# On Variation of Product Cordial Labeling of Subdivision of Flower and Its Path Union 

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#### Abstract

In this paper, we prove that the subdivision of Flower graph and the Path union of $k$ copies of subdivision of Flower graphs are Product cordial graph and Total product cordial graphs. We also extend to prove that the path union from the outer vertex of the subdivision of Flower admits Product cordial and total product cordial labeling.


$\underline{\text { Keywords- Product cordial labeling, Total product cordial labeling, Flower graph, Subdivision, Path union. }}$

## I. INTRODUCTION

In mathematical discipline of graph theory, a graph labeling is the assignment of labels, traditionally represented by integers, to the edges or vertices, or both, of a graph subject to certain constraints. Graph labeling is one of the most interesting concepts in Graph theory and it was introduced by Rosa [3] in 1960's. Graph labeling draws an effective communication between Number Theory and in analysing the structure of the graphs. Cordial labeling was first introduced by Cahit [1] in 1987. In 2004, Sundaram et al[4] introduced the notion of Product cordial labeling .Sundaram et al [4] proved the various graphs are product cordial graphs such as tree graphs, Triangular snakes, Dragon graphs, Helm, $P_{m} \cup P_{n}, C_{m} \cup P_{n}, W_{m} \cup C_{m}$. In 2006, Sundaram et al [5] introduced the notion of Total product cordial labeling. Sundaram et al [5] proved the various graphs are Total product cordial graphs such as Tree graphs, all cycles except $C_{4}, K_{n, 2 n-1}, C_{n}$, Wheels, Helms. Let the Flower graph $F l_{n}, n \geq 3$, is the graph obtained from the Helm graph $H_{m}$ by attaching each pendant vertex to the apex of the wheel graph $w_{n}$ and subdividing each edge by a vertex is called Subdivision of Flower graph. For a detailed survey on Total product cordial graphs one can refer to Gallian [2].

## II. MAIN RESULTS

## Theorem 2.1:

Subdivision of Flower graph admits Product cordial labeling.

## Proof:

Let the Flower graph $S\left(F l_{n}\right), n \geq 3$ is the graph obtained from the Helm $H_{m}$ by attaching an edge from each pendant vertex to the apex of the wheel $W_{n}$ and subdividing each edge by a vertex.

We denotes the vertices of $S\left(F l_{n}\right), n \geq 3$ as follows:
Let $u$ denotes the apex vertex. Let $w_{1}, w_{2}, \ldots, w_{n}$ denotes the vertices obtained by subdividing the edges $u u_{i}$. Let $u_{1}, u_{2}, \ldots, u_{n}$ denotes the vertices of the cycle of Flower graph. Let $u_{i} u_{i+1}(1 \leq i \leq n)$ subdivides as the vertices $y_{i}$ on the cycle $c_{n}$. Let $v_{1}, v_{2}, \ldots, v_{n}$ denotes the end vertices of the $S\left(F l_{n}\right)$. Let $x_{1}, x_{2}, \ldots, x_{n}$ denotes the vertices which is obtained by subdividing the $u v_{i}$ edges of Flower graph. Let $z_{1}, z_{2}, \ldots, z_{n}$ denotes the vertices obtained by subdividing the edges $u_{i} v_{i}$ of Flower graph.


Figure: 2.1 Subdivision of Flower graph
Let $V\left(S\left(F l_{n}\right)\right)=\left\{u, w_{i}, u_{i}, z_{i}, v_{i}, x_{i}, y_{i} ;(1 \leq i \leq n)\right\}$
Let

$$
E\left(S\left(F l_{n}\right)\right)=
$$

$\left\{\left(u w_{i}, w_{i} u_{i}\right) \cup\left(u_{i} z_{i}, z_{i} v_{i}\right) \cup\left(u_{i} y_{i}, y_{i} u_{i+1}\right) \cup\right.$
$\left.\left(u x_{i}, x_{i} v_{i}\right) ;(1 \leq i \leq n)\right\}$
The total number of vertices in $S\left(F l_{n}\right)$ is $6 n+1$ and the total number of edges in $S\left(F l_{n}\right)$ is $8 n(n \geq 3)$.

The vertex labels for the subdivision of Flower graph are defined below:

$$
\begin{gather*}
f: V\left(S\left(F l_{n}\right) \rightarrow\{0,1\}\right. \\
f\left(u, w_{i}, u_{i}, y_{i}\right)=1 \tag{1}
\end{gather*}
$$

$f\left(z_{i}, v_{i}, x_{i}\right)=0$ where $(1 \leq i \leq n)$
The edge labels for the subdivision of Flower graph are defined
below:

$$
f\left(u w_{i}, w_{i} u_{i}, u_{i} y_{i}, y_{i} u_{i+1}\right)=1
$$

$$
\begin{equation*}
f\left(u x_{i}, x_{i} v_{i}, u_{i} z_{i}, z_{i} v_{i}\right)=0 \tag{2}
\end{equation*}
$$

Using the above equations, the vertex and edge labels are computed as follows:
The number of vertices with label ' 0 ' $=v_{f}(0)=\frac{6 n}{2}$
The number of vertices with label ' 1 ' $=v_{f}(1)=\frac{6 n}{2}+1$
The number of edges with label $\quad 0^{\prime} \quad=e_{f}(0)=\frac{8 n}{2}$
The number of edges with label $\quad 1^{\prime}=e_{f}(1)=\frac{8 n}{2}$
Let $m$ denotes the number of petals in Flower graph.

## Case: $\boldsymbol{m}$ is odd

$$
\begin{aligned}
& \left|v_{f}(0)-v_{f}(1)\right|=\left|\frac{6 n}{2}+1-\frac{6 n}{2}\right|=1 \\
& \left|e_{f}(0)-e_{f}(1)\right|= \\
& \left|\frac{8 n}{2}-\frac{8 n}{2}\right|=0
\end{aligned}
$$

The condition for Product cordial labeling is,

$$
\left|v_{f}(0)-v_{f}(1)\right| \leq 1
$$

$\left|e_{f}(0)-e_{f}(1)\right| \leq 1$
Thus, the condition is satisfied.

## Case: $m$ is even

$$
\begin{array}{r}
\left|v_{f}(0)-v_{f}(1)\right|=\left|\frac{6 n}{2}+1-\frac{6 n}{2}\right|=1 \\
\left|e_{f}(0)-e_{f}(1)\right|=
\end{array}
$$

$$
\left|\frac{8 n}{2}-\frac{8 n}{2}\right| \quad=0
$$

Therefore, $\left|v_{f}(0)-v_{f}(1)\right| \leq 1 \quad$ and $\quad\left|e_{f}(0)-e_{f}(1)\right| \leq$ 1
Thus, the condition is satisfied.
Hence the subdivision of Flower graph admits product cordial labeling.

## Theorem 2.2:

Subdivision of Flower graph admits Total product cordial labeling.

## Proof:

Denote the vertices and edges of $S\left(F l_{n}\right)$ as follows:
Let $V\left(S\left(F l_{n}\right)\right)=\left\{u, w_{i}, u_{i}, z_{i}, v_{i}, x_{i}, y_{i} ;(1 \leq i \leq n)\right\}$
Let $\quad E\left(S\left(F l_{n}\right)\right)=$
$\left\{\left(u w_{i}, w_{i} u_{i}\right) \cup\left(u_{i} z_{i}, z_{i} v_{i}\right) \cup\left(u_{i} y_{i}, y_{i} u_{i+1}\right) \cup\right.$
$\left.\left(u x_{i}, x_{i} v_{i}\right) ;(1 \leq i \leq n)\right\}$
The total number of vertices in $S\left(F l_{n}\right)$ is $6 n+1$ and the total number of edges in $S\left(F l_{n}\right)$ is $8 n(n \geq 3)$.
The vertex labels for the subdivision of Flower graph are defined below:

$$
\begin{gather*}
f: V\left(S\left(F l_{n}\right) \rightarrow\{0,1\}\right. \\
f\left(u, w_{i}, u_{i}, y_{i}\right)=1 \tag{3}
\end{gather*}
$$

$f\left(z_{i}, v_{i}, x_{i}\right)=0$ where $(1 \leq i \leq n)$
The edge labels for the subdivision of Flower graph are defined below:

$$
\begin{gather*}
f: E\left(S\left(F l_{n}\right) \rightarrow\{0,1\}\right. \\
f\left(u w_{i}, w_{i} u_{i}, u_{i} y_{i}, y_{i} u_{i+1}\right)=1 \\
f\left(u x_{i}, x_{i} v_{i}, u_{i} z_{i}, z_{i} v_{i}\right)=0 \tag{4}
\end{gather*}
$$

Using the above equations, the vertex and edge labels are computed as follows:
The number of vertices with label ${ }^{\prime} 0$ ' $=v_{f}(0)=\frac{6 n}{2}$
The number of vertices with label ' 1 ' $=v_{f}(1)=\frac{6 n}{2}+1$
The number of edges with label $\quad 0^{\prime} \quad=e_{f}(0)=\frac{8 n}{2}$
The number of edges with label $\quad 1$ ' $=e_{f}(1)=\frac{8 n}{2}$
The condition for total product cordial labeling is,

$$
\left|v_{f}(0)+e_{f}(0)-\left(v_{f}(1)+e_{f}(1)\right)\right| \leq 1
$$

Let $m$ denotes the number of petals in Flower graph.
Case: $\boldsymbol{m}$ is odd

$$
\left|\frac{6 n}{2}+\frac{8 n}{2}-\left(\frac{6 n}{2}+1+\frac{8 n}{2}\right)\right|=1
$$

Thus, the condition is satisfied.

## Case: $\boldsymbol{m}$ is even

$$
\left|\frac{6 n}{2}+\frac{8 n}{2}-\left(\frac{6 n}{2}+1+\frac{8 n}{2}\right)\right|=1
$$

Thus, the condition is satisfied.
Hence the subdivision of Flower graph admits Total product cordial labeling.

## Theorem 2.3:

Path union of $k$ copies of Subdivision of Flower graph admits Product cordial labeling.

## Proof:

Let the Flower graph $S\left(F l_{n}\right), n \geq 3$ is the graph obtained from the Helm $H_{m}$ by attaching an edge from each pendant vertex to the apex of the wheel $W_{n}$ and subdividing each edge by a vertex.
We denotes the vertices of $S\left(F l_{n}\right)$ as follows:
Let $u_{1}^{\prime}, u_{2}^{\prime}, \ldots, u_{n}^{\prime}$ denotes the apex vertices. Let $w_{1 i}, w_{2 i}, \ldots, w_{n i}$ denotes the vertices i.e., obtained by subdividing the edges $u_{n}^{\prime} u_{n i}$. Let $u_{1 i}, u_{2 i}, \ldots, u_{n i}$ denotes the vertices of the cycle of Flower graph. Let $u_{i} u_{i+1}(1 \leq i \leq$ $n$ ) subdivides as the vertices $y_{1 i}, y_{2 i}, \ldots, y_{n i}$ on the cycle $c_{n}$. Let $v_{1 i}, v_{2 i}, \ldots, v_{n i}$ denotes the end vertices of the $S\left(F l_{n}\right)$. Let $x_{1 i}, x_{2 i}, \ldots, x_{n i}$ denotes the vertices i.e., obtained by subdividing the $u_{n}^{\prime} v_{n i}$ edges of Flower graph. Let $z_{1 i}, z_{2 i}, \ldots, z_{n i}$ denotes the vertices obtained by subdividing the edges $u_{n i} v_{n i},(n=1,2 .$.$) of Flower graph. Let P_{i}^{\prime}$ denotes the number of vertices in the path. Let $p_{i}$ denotes the edges in the path.
Here $u_{n}^{\prime}=P_{i}^{\prime}$

Let $V\left(S\left(F l_{n}\right)\right)=\left\{u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}, P_{i}^{\prime} ;(1 \leq i \leq\right.$ n) $\}$

Let $\quad E\left(S\left(F l_{n}\right)\right)=\left\{\left(u_{n}^{\prime} w_{n i}, w_{n i} u_{n i}\right) \cup\left(u_{n i} z_{n i}, z_{n i} v_{n i}\right) \cup\right.$ $\left.\left(u_{n i} y_{n i}, y_{n i} u_{n(i+1)}\right) \cup\left(u_{n}^{\prime} x_{n i}, x_{n i} v_{n i}\right) ;(1 \leq i \leq n)\right\}$


Figure 2.3 Subdivision of Flowers are connected by the path from the apex.

The total number of vertices in $S\left(F l_{n}\right)$ is $(6 n+1) k$ 's and the total number of edges in $S\left(F l_{n}\right)$ is ( $\left.8 n\right) k^{\prime} s+p_{i}(n \geq 3)$ Let $k$ denotes the copies of subdivision of Flower graph.
Let ' $n$ ' denotes the vertex labels for the subdivision of Flower graph is defined below:

## Case: $\boldsymbol{n}$ is even

$f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)$
$=\left\{\begin{array}{c}1,\left[\frac{k(6 n+1)}{2}\right] \\ 0, \text { otherwise }\end{array}\right.$

## Case: $\boldsymbol{n}$ is odd

$f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, y_{n i}\right)=1$
$f\left(z_{n i}, v_{n i}, x_{n i}\right)=0$
$f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)=\left\{\begin{array}{c}1,\left[\frac{k-1}{2}(6 n+1)\right] \\ 0,\left[\frac{k-1}{2}(6 n+1)\right] \\ 1,0 ;\left[\frac{(6 n+1)}{2}\right]\end{array}\right.$
(6)

Using the above equations, the vertex and edge labels are computed as follows:
The number of vertices with label ' 0 ' $=v_{f}(0)=\frac{6 n}{2}$
The number of vertices with label ' 1 ' $=v_{f}(1)=\frac{6 n}{2}+1$

The number of edges with label ' 0 ' $=e_{f}(0)=\frac{8 n}{2}+p_{i}$
The number of edges with label' 1 ' $=e_{f}(1)=\frac{8 n}{2}+p_{i}$
Case: $\boldsymbol{k}$ is odd

$$
\begin{aligned}
&\left|v_{f}(0)-v_{f}(1)\right|=\left|\frac{6 n}{2}-\frac{6 n}{2}-1\right| \\
&\left|e_{f}(0)-e_{f}(1)\right|=1 \\
&\left|\frac{8 n}{2}+p_{i}-\frac{8 n}{2}+p_{i}\right|=0
\end{aligned}
$$

The condition for Product cordial labeling is,

$$
\left|v_{f}(0)-v_{f}(1)\right| \leq 1 \text { and } \mid e_{f}(0)-
$$

$e_{f}(1) \mid \leq 1$
Thus, the condition is satisfied.

## Case: $k$ is even

$$
\left|v_{f}(0)-v_{f}(1)\right|=\left|\frac{6 n}{2}-\frac{6 n}{2}-1\right|=0
$$

$$
\left|\frac{8 n}{2}+p_{i}-\frac{8 n}{2}-p_{i}\right|=1 \quad\left|e_{f}(0)-e_{f}(1)\right|=
$$

Therefore, $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$
Thus, the condition is satisfied.
Hence the path union of $k$ copies of subdivision of Flower graph admits product cordial labeling.

## Theorem 2.4:

Path union of $k$ copies Subdivision of Flower graph admits Total product cordial labeling.

## Proof:

Denotes the vertex and edges of path union of $k$ copies of $S\left(F l_{n}\right)$ as follows:
Let $V\left(S\left(F l_{n}\right)\right)=\left\{u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}, P_{i}^{\prime} ;(1 \leq i \leq\right.$ n) $\}$

Let $E\left(S\left(F l_{n}\right)\right)=\left\{\left(u_{n}^{\prime} w_{n i}, w_{n i} u_{n i}\right) \cup\left(u_{n i} z_{n i}, z_{n i} v_{n i}\right) \cup\right.$ $\left.\left(u_{n i} y_{n i}, y_{n i} u_{n(i+1)}\right) \cup\left(u_{n}^{\prime} x_{n i}, x_{n i} v_{n i}\right) ;(1 \leq i \leq n)\right\}$
The total number of vertices in $S\left(F l_{n}\right)$ is $(6 n+1) k^{\prime} s$ and the total number of edges in $S\left(F l_{n}\right)$ is ( $\left.8 n\right) k^{\prime} s+p_{i}(n \geq 3)$
Let $k$ denotes the copies of subdivision of Flower graph.
Let ' $n$ ' denotes the vertex labels for the subdivision of Flower graph is defined below:

## Case: $\boldsymbol{n}$ is even

$f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)$
$=\left\{\begin{array}{c}1,\left[\frac{k(6 n+1)}{2}\right] \\ 0, \text { otherwise }\end{array}\right.$
Case: $\boldsymbol{n}$ is odd

$$
f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, y_{n i}\right)=1
$$

$$
f\left(z_{n i}, v_{n i}, x_{n i}\right)=0
$$

$f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)=\left\{\begin{array}{c}1,\left[\frac{k-1}{2}(6 n+1)\right] \\ 0,\left[\frac{k-1}{2}(6 n+1)\right] \\ 1,0 ;\left[\frac{(6 n+1)}{2}\right]\end{array}\right.$
(8)

Using the above equations, the vertex and edge labels are computed as follows:
The number of vertices with label ${ }^{\prime} 0^{\prime}=v_{f}(0)=\frac{6 n}{2}$
The number of vertices with label ' 1 ' $=v_{f}(1)=\frac{6 n}{2}+1$

The number of edges with label ' 0 ' $=e_{f}(0)=\frac{8 n}{2}+p_{i}$
The number of edges with label' 1 ' $=e_{f}(1)=\frac{8 n}{2}+p_{i}$
The condition for total product cordial labeling is,

$$
\left|v_{f}(0)+e_{f}(0)-\left(v_{f}(1)+e_{f}(1)\right)\right| \leq 1
$$

## Case: $\boldsymbol{k}$ is odd

$$
\left|\frac{6 n}{2}+\left(\frac{8 n+p_{i}}{2}\right) k-\left(\frac{6 n}{2}+1+\left(\frac{8 n+p_{i}}{2}\right) k\right)\right|=1
$$

Thus, the condition is satisfied.

## Case: $\boldsymbol{k}$ is even

$$
\left|\frac{6 n}{2}+\left(\frac{8 n+p_{i}}{2}\right) k-\left(\frac{6 n}{2}+1+\left(\frac{8 n+p_{i}}{2}\right) k\right)\right|=1
$$

Thus, the condition is satisfied.
Hence the path union of $k$ copies of subdivision of Flower graph admits Total product cordial labeling.

## Theorem 2.5:

Path union of $k$ copies of Subdivision of Flower graph admits Product cordial labeling when $k$ is even.

## Proof:

Let the Flower graph $S\left(F l_{n}\right), n \geq 3$ is the graph obtained from the Helm $H_{m}$ by attaching an edge from each pendant vertex to the apex of the wheel $W_{n}$ and subdividing each edge by a vertex.
We denotes the vertices of $S\left(F l_{n}\right)$ as follows:
Let $u_{1}^{\prime}, u_{2}^{\prime}, \ldots, u_{n}^{\prime}$ denotes the apex vertices. Let $w_{1 i}, w_{2 i}, \ldots, w_{n i}$ denotes the vertices i.e., obtained by subdividing the edges $u_{n}^{\prime} u_{n i}$. Let $u_{1 i}, u_{2 i}, \ldots, u_{n i}$ denotes the vertices of the cycle of Flower graph. Let $u_{i} u_{i+1}(1 \leq i \leq$ $n$ ) subdivides as the vertices $y_{1 i}, y_{2 i}, \ldots, y_{n i}$ on the cycle $c_{n}$. Let $v_{1 i}, v_{2 i}, \ldots, v_{n i}$ denotes the end vertices of the $S\left(F l_{n}\right)$. Let $x_{1 i}, x_{2 i}, \ldots, x_{n i}$ denotes the vertices i.e., obtained by subdividing the $u_{n}^{\prime} v_{n i}$ edges of Flower graph. Let $z_{1}, z_{2}, \ldots, z_{n}$ denotes the vertices obtained by subdividing the edges $u_{n i} v_{n i},(n=1,2 .$.$) of Flower graph. Let P_{i}^{\prime}$ denotes the number of vertices in the path. Let $p_{i}$ denotes the edges in the path.
Here $v_{n i}=P_{i}^{\prime}$
Let $V\left(S\left(F l_{n}\right)\right)=\left\{u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}, P_{i}^{\prime} ;(1 \leq i \leq\right.$ n) $\}$

Let $\quad E\left(S\left(F l_{n}\right)\right)=\left\{\left(u_{n}^{\prime} w_{n i}, w_{n i} u_{n i}\right) \cup\left(u_{n i} z_{n i}, z_{n i} v_{n i}\right) \cup\right.$ $\left.\left(u_{n i} y_{n i}, y_{n i} u_{n(i+1)}\right) \cup\left(u_{n}^{\prime} x_{n i}, x_{n i} v_{n i}\right) ;(1 \leq i \leq n)\right\}$


Figure: 2.5 Subdivision of Flowers are connected by the path from the outer vertex.

Let $k$ denotes the copies subdivision of Flower graph.
The total number of vertices in $S\left(F l_{n}\right)$ is $(6 n+1) k^{\prime} s$ and the total number of edges in $S\left(F l_{n}\right)$ is $(8 n) k^{\prime} s+p_{i}(n \geq 3)$. Let ' $n$ ' denotes the vertex labels for the subdivision of Flower graph is given below:

## Case: $\boldsymbol{n}$ is even

$f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)=$
$\left\{\begin{array}{l}1,\left[\frac{k(6 n+1)}{2}\right]\end{array}\right.$

$$
\begin{gather*}
f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, y_{n i}\right)=1 \\
f\left(z_{n i}, v_{n i}, x_{n i}\right)=0 \\
f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)=\left\{\begin{array}{c}
1,\left[\frac{k-1}{2}(6 n+1)\right] \\
0,\left[\frac{k-1}{2}(6 n+1)\right] \\
1,0 ;\left[\frac{(6 n+1)}{2}\right]
\end{array}\right. \tag{10}
\end{gather*}
$$

Using the above equations, the vertex and edge labels are computed as follows:
The number of vertices with label ${ }^{\prime} 0^{\prime}=v_{f}(0)=\frac{6 n}{2}$
The number of vertices with label ' 1 ' $=v_{f}(1)=\frac{6 n}{2}+1$
The number of edges with label ' 0 ' $=e_{f}(0)=\frac{8 n}{2}+p_{i}$
The number of edges with label ' 1 ' $=e_{f}(1)=\frac{8 n}{2}+p_{i}$
The condition for Product cordial labeling is,

$$
\begin{aligned}
& \left|v_{f}(0)-v_{f}(1)\right| \leq 1 \\
& \left|e_{f}(0)-e_{f}(1)\right| \leq 1
\end{aligned}
$$

## When $k$ is even

$$
\begin{array}{r}
\left|v_{f}(0)-v_{f}(1)\right|=\left|\frac{6 n}{2}-\frac{6 n}{2}-1\right|=0 \\
\left|e_{f}(0)-e_{f}(1)\right|=\left\lvert\,\left(\frac{8 n}{2}+\right.\right.
\end{array}
$$

$\left.p_{i}\right) \left.k-\left(\frac{8 n}{2}+p_{i}\right) k \right\rvert\,=1$
Therefore, $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$
Thus, the condition is satisfied.
Hence the path union $k$ copies of subdivisions of Flower graph admits product cordial labeling when $k$ is even.

## Theorem 2.6:

Path union of $k$ copies of Subdivision of Flower graph admits Total product cordial labeling when $k$ is even.

## Proof:

Denotes the vertex and edges of Path union of even number $S\left(F l_{n}\right)$ as follows:
Let $V\left(S\left(F l_{n}\right)\right)=\left\{u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}, P_{i}^{\prime} ;(1 \leq i \leq\right.$ n) $\}$

Let $E\left(S\left(F l_{n}\right)\right)=\left\{\left(u_{n}^{\prime} w_{n i}, w_{n i} u_{n i}\right) \cup\left(u_{n i} z_{n i}, z_{n i} v_{n i}\right) \cup\right.$ $\left.\left(u_{n i} y_{n i}, y_{n i} u_{n(i+1)}\right) \cup\left(u_{n}^{\prime} x_{n i}, x_{n i} v_{n i}\right) ;(1 \leq i \leq n)\right\}$
Let $k$ denotes the copies of subdivision of Flower graph.
The total number of vertices in $S\left(F l_{n}\right)$ is $(6 n+1) k ' s$ and the total number of edges in $S\left(F l_{n}\right)$ is ( $\left.8 n\right) k^{\prime} s+p_{i}(n \geq 3)$.
Let ' $n$ ' denotes the total number of vertices for the subdivision of Flower graph is given below:

## Case: $\boldsymbol{n}$ is even

$f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)=$
$\left\{\begin{array}{l}1,\left[\frac{k(6 n+1)}{2}\right] \\ 0, \text { otherwise }\end{array}\right.$
Case: $\boldsymbol{n}$ is odd

$$
\begin{gathered}
f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, y_{n i}\right)=1 \\
f\left(z_{n i}, v_{n i}, x_{n i}\right)=0
\end{gathered}
$$

## Case: $\boldsymbol{n}$ is odd

$$
f\left(u_{n}^{\prime}, w_{n i}, u_{n i}, z_{n i}, v_{n i}, x_{n i}, y_{n i}\right)=\left\{\begin{array}{c}
1,\left[\frac{k-1}{2}(6 n+1)\right]  \tag{12}\\
0,\left[\frac{k-1}{2}(6 n+1)\right] \\
1,0 ;\left[\frac{(6 n+1)}{2}\right]
\end{array}\right.
$$

Using the above equations, the vertex and edge labels are computed as follows:
The number of vertices with label ' 0 ' $=v_{f}(0)=\frac{6 n}{2}$
The number of vertices with label ' 1 ' $=v_{f}(1)=\frac{6 n}{2}+1$
The number of edges with label ' 0 ' $=e_{f}(0)=\frac{8 n}{2}+p_{i}$
The number of edges with label ' 1 ' $=e_{f}(1)=\frac{8 n}{2}+p_{i}$

The condition for total product cordial labeling is,

$$
\left|v_{f}(0)+e_{f}(0)-\left(v_{f}(1)+e_{f}(1)\right)\right| \leq 1
$$

## When $\boldsymbol{k}$ is even

$$
\left|\frac{6 n}{2}+\left(\frac{8 n+p_{i}}{2}\right) k-\left(\frac{6 n}{2}+1+\left(\frac{8 n+p_{i}}{2}\right) k\right)\right|=1
$$

Thus, the condition is satisfied.
Hence the path union of $k$ copies of subdivisions of Flower graph admits Total product cordial labeling when $k$ is even.

## III. CONCLUSION

In this paper, we have proved that the Product cordial labeling and total product cordial labeling of subdivision of Flower graph and the path union of subdivision of Flower graph admits Product cordial labeling and total product cordial labeling. Also, we can prove for any other subdivisions of graphs are Product cordial and Total product cordial labeling.

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