Modification of BM3D Algorithm for Representing Volumetric Data on Medical Images

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Abstract: In the past decade, sufficient powerful Denoising algorithms have been devised - among them the patch-based nonlocal schemes, such as BM3D, have shown outstanding performance. The BM3D employs a non-local modeling of images by collecting similar image patches in 3D arrays. The so-called collaborative filtering applied on such a 3D array is realized by transform-domain shrinkage. The block-matching with 3D transform domain collaborative filtering (BM3D) achieves very good performance in image Denoising. However, BM3D becomes ineffective when an image is heavily contaminated by noise. This is because it allows block-matching to search out of the region where a template block is located, resulting in poor matching. To address this, paper proposes an approach that is an extension of BM3D to represent to volumetric data & image reconstruction.

Keywords: Modified BM3D, Volumetric Data, Image reconstruction

I. INTRODUCTION

In NLmeans, the basic idea is to build a point wise estimate of the image where each pixel is obtained as a weighted average of pixels centered at regions that are similar to the region centered at the estimated pixel. The estimates are nonlocal because, in principle, the averages can be calculated over all pixels of the image. One of the most powerful and effective extensions of the nonlocal filtering approach is the grouping and collaborative filtering paradigm embodied by the BM3D image Denoising algorithm [2]. This algorithm is based on an enhanced sparse representation in transform domain. The enhancement of the sparsity is achieved by grouping similar 2-D fragments of the image into 3-D data arrays which are called “group”. Such groups are processed through a special procedure, named collaborative filtering, which consists of three successive steps: firstly a 3-D transformation is applied to the group, secondly the transformed group coefficients are shrunk, and finally a 3-D group estimate is obtained by inverting the 3-D transformation. Due to the similarity between the grouped fragments, the noise can be well separated by shrinkage because the 3-D transformation discloses a highly sparse representation of the true signal in transform domain. In this way, the collaborative filtering reveals even the finest details shared by the jointly filtered 2-D fragments preserving at the same time their essential unique features.

The BM3D algorithm presented in [2] represents the current state of the art in 2-D image Denoising, demonstrating a performance significantly superior to that of all previously existing methods. Recent works discuss the near-optimality of this approach and offer further insights about the rationale of the algorithm [3], [4].

II. PROPOSED MBM3D ALGORITHM

The proposed Modified BM3D(MBM3D) algorithm utilize cubes of voxels. The group formed by stacking mutually similar cubes is hence a four-dimensional orthope (hyper rectangle) whose fourth dimension, along which the cubes are stacked, embodies the nonlocal correlation across the data. As in BM3D[2][9]the spectrum of the group is highly sparse, leading to a very effective separation of signal and noise by either thresholding or Wiener filtering. After inverse transformation, we obtain the estimates of each grouped cube, which are then aggregated at their original locations using adaptive weights Modified BM3D as a regularizer operator for the reconstruction of incomplete volumetric data.

Reconstruction procedure works iteratively. In each iteration the missing part of the spectrum is excited with random noise; then, after transforming the excited spectrum to the voxel domain, the modified BM3D filter attenuates the noise present in both magnitude and phase of the data, thus disclosing even the faintest details from the
incomplete and degraded observations. The overall procedure can be interpreted as a progressive approximation in which the denoising filter directs the stochastic search towards the solution.

Algorithm 1: **Hard Thresholding Estimate**

Input: Original Image, Noisy Image  
Output: Basic Estimate of Denoised images are obtained

Method  
For each voxels in the noisy image do the following  
- Block match grouping to find similar 4D array  
- Hard thresholding - the four-dimensional groups are formed by stacking together, along an additional fourth dimension, (three-dimensional) noisy cubes  
- Inverse 4D transform Domain  
- Aggregate to reconstruct the image  

Algorithm 2: **Wiener Filtering Estimate**

Input: Hard threshold Image, Noisy Image  
Output: Wiener Filtering Estimate of Denoised images are obtained

Method  
For each voxels in the noisy image do the following  
- Block match grouping to find similar 4D array  
- Wiener Filtering Estimate  
- Inverse 4D transform Domain  
- Aggregate to reconstruct the image  

**III. EXPERIMENTAL PROCEDURE**

Consider a noisy image \( z : X \rightarrow \mathbb{R} \) of the form
\[
z(x) = y(x) + \eta(x), \quad x \in X,
\]
where \( y \) is the original, unknown, volumetric signal, \( x \) is a 3-D coordinate belonging to the signal domain \( X \subseteq \mathbb{Z}^3 \), and \( \eta(x) \) is independent and identically distributed (i.i.d.) Gaussian noise with zero mean and known standard deviation.

**3.1 Methodology for Hard-thresholding & Weiner Estimate**

**Hard-thresholding stage**: Let \( C_{z \times R} \) denote a cube of \( L \times L \times L \) voxels, with \( L \in \mathbb{N} \), extracted from \( z \) at the 3-D coordinate \( x \times R \), which identifies its top-left-front corner. In the hard-thresholding stage, the four-dimensional groups are formed by stacking together, along an additional fourth dimension, (three-dimensional) noisy cubes similar to \( C_{z \times R} \). Specifically, the similarity between two cubes is measured via the photometric distance \( d \).
In the grouping step, a group consisting of mutually similar cubes extracted from \( z \) is built for every (reference) cube \( C_z \times R \). Two cubes are considered similar if their distance (2) is smaller than or equal to a predefined threshold \( \tau_{\text{match}} \) match which thus controls the minimum accepted cube-similarity. Formally, we first define a set containing the indices of the cubes similar to \( C_z \times R \) as

\[
S_{zR}^x = \{ x \in X; d(C_{zR}^x, C_{zR}^x) \leq \tau_{\text{match}} \}
\]  

(3)

transform, which we denote as a joint four-dimensional transform \( T_{4D} \), are separately applied to every dimension of the group (3). The so-obtained 4-D group spectrum is then shrunk coefficient by coefficient by a hard-thresholding operator

\[
y^{ht}(\tau^{ht}_{4D}(G^{ht}_{S^x}))
\]  

(4)

Step 2: Cube Matching and Weiner filtering

In the Wiener-filtering stage, the grouping is performed within the basic estimate \( \hat{Y}^{ht} \). We expect the obtain a more accurate and reliable matching because the noise level in \( \hat{Y}^{ht} \) is considerably smaller than that in \( z \).

\[
S_{zR}^x = \{ x \in X; d(C_{zR}^x, C_{zR}^x) \leq \tau_{\text{match}} \}
\]  

(5)

where \( d(\cdot) \) is defined as in (2).

The collaborative filtering is implemented as an empirical Wiener filter. Analogously to (4), at first a group \( G^{ht}_{S^x} \) is extracted from \( \hat{Y}^{ht} \) using the set of coordinates (5), then from the energy of its spectrum we define the empirical Wiener filter coefficients as

\[
W_{S^x}^{ht} = \frac{|r^{wie}_{4D}(G^{ht}_{S^x})|^2}{|r^{wie}_{4D}(G^{ht}_{S^x})|^2 + \sigma^2}
\]  

(6)

where \( \sigma \) denotes the standard deviation of the noise, and \( r^{wie}_{4D} \) is a transform operator composed by four 1-D linear transformations which are in general different than those in \( r^{tie}_{4D} \). Subsequently, we use the same set (5) to extract a second (noisy) group, termed \( G^{ht}_{S^x} \), from the observation \( z \).

The coefficients shrinkage is implemented as element-by-element multiplication between the spectrum of the noisy group and the Wiener-filter coefficients (6). The estimate of the group

\[
\hat{G}_{S^x}^{ht} = \tau_{4D}^{-1}(W_{S^x}^{ht}) \hat{G}_{S^x}^{ht}(G_{S^x}^{ht})
\]  

(7)

is finally produced by applying the inverse four-dimensional transform \( r_{4D}^{-1} \) to the shrunk spectrum. The final estimate \( \hat{Y}^{wie} \) is produced through a convex combination, analogous to (3), in which the sets (3) are replaced with (5), and the aggregation weights for a specific group estimate (7) are defined from the energy of the Wiener-filter coefficients (6) as

\[
\omega^{wie}_{S^x} = \sigma^{-2} \left| W_{S^x}^{ht} \right|^2
\]  

(8)

3.2 Methodology for Image Reconstruction

In this section, an iterative procedure is proposed, designed for the joint Denoising and reconstruction of incomplete volumetric data, that uses the proposed MBM3D algorithm as a regularizer operator.

A. Observation Model

The observation model for the volumetric reconstruction problem is given by

\[
\theta = \tau(y^e^{i\varphi}) + \eta
\]  

(9)

where is the transform-domain representations of the unknown volumetric data having magnitude \( y : X \rightarrow R^l \) and absolute (unwrapped) phase \( 0 : X \rightarrow Z^l \). \( i \) is the imaginary unit, \( T \) is for our purposes, the Fourier transform, and \( \eta(\cdot) \equiv (0, \sigma^2) \) i.i.d. complex Gaussian noise with zero mean and standard deviation.

Let \( \Omega \) be the support of the available portion of the spectrum \( \theta \). We define a sampling operator \( S \) as the characteristic (indicator) function, which is 1 over \( \Omega \) and 0 elsewhere. By means of \( S \), we can split the spectrum in two complementary parts as

\[
\theta = S \theta + (1 - S) \theta
\]  

(10)

IV. RESULTS & DISCUSSION

In this section we present and discuss the experimental results obtained by the MBM3D using noisy magnetic resonance phantoms, because we recognize medical imaging to be one of the most prominent applications based on volumetric data. We measure the objective quality of the Denoising through its

\[
PSNR(y, \hat{y}) = 10 \log_{10} \left( \frac{D^2|X|}{\sum_{x \in X} (y(x) - \hat{y}(x))^2} \right)
\]  

(11)

where \( D \) is the peak of \( y \times x \in X : y(x) > 10 \_ D=255g \). We also evaluate our experiments with the structure similarity index (SSIM), that is a metric originally presented for 2-D.
images in and extended to 3-D data in that better relates to the human visual system than traditional methods based on the mean squared error such as the PSNR. The test data of our experiment is the BrainWeb and 3-D Shepp-Logan phantom of size 128x128x128 voxels; cross-sections of both original phantoms are shown in Fig 2 to Fig 6.

Table I Parameters setting for the proposed MBM3D Algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hard thresholding</th>
<th>Weiner filtering</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Modified</td>
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<tr>
<td>Cube size L</td>
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<td>32</td>
</tr>
<tr>
<td>Group size M</td>
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<td></td>
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<tr>
<td>Step N</td>
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<tr>
<td>Similar th. τmatch</td>
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<td>24.6</td>
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<tr>
<td>Shrinkage thr. λ</td>
<td>2.7</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Iteration #0001 (3.8sec) - Excitation sigma 5.00%
Magnitude PSNR 25.18dB SSIM 0.77 - Phase PSNR 18.42dB SSIM 0.82
RADIAL subsampling 29.11% - Initial sigma 5.00%

Phantom size 128x128x128, displaying cross-section 64 of 128

Fig 2: Radial Trajectory of brainweb phantom

Iteration #0001 (3.8sec) - Excitation sigma 5.00%
Magnitude PSNR 12.42dB SSIM 0.24 - Phase PSNR 11.21dB SSIM 0.24
SPIRAL subsampling 29.08% - Initial sigma 5.00%

Phantom size 128x128x128, displaying cross-section 64 of 128
Fig 3: Spiral Trajectory of brain web phantom

Iteration #0001 (3.8sec) - Excitation sigma 5.00%
Magnitude PSNR 26.74dB SSIM 0.79 - Phase PSNR 18.92dB SSIM 0.82
LOG SPIRAL subsampling 30.30% - Initial sigma 5.00%

Phantom size 128x128x128, displaying cross-section 64 of 128

Fig 4: Logarithmic spiral trajectory of brain web phantom

Iteration #0001 (3.7sec) - Excitation sigma 5.00%
Magnitude PSNR 14.37dB SSIM 0.28 - Phase PSNR 11.92dB SSIM 0.25
LIM ANGLE subsampling 29.22% - Initial sigma 5.00%

Phantom size 128x128x128, displaying cross-section 64 of 128
V. CONCLUSION

MBM3D is an extension of BM3D for 3D images (Medical) used for volumetric data. The proposed MBMD filter significantly outperforms the current state of the art in volumetric data denoising. In particular, the Denoising performance on MR images corrupted by Gaussian noise demonstrates the superiority of the proposed approach in terms of both objective (PSNR and SSIM) and subjective visual quality. MBMD has been also successfully tested on the Denoising of real MRI data.

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