Performance Analysis of a Gold extraction process system under Different Failure using Boolean algebra

A. Chandra1*, S. Gupta2

1Amity Institute of Microbial Technology, Amity University, Noida, India
2Amity institute of Applied Science, Amity University, Noida, India

*Corresponding Author: achandra@amity.edu

Available online at: www.ijcseonline.org

Abstract—This paper deals with the ensuring reliability and Availability of Gold extraction system is extremely complex and extends to all the stages of the service life of a system. The gold extraction system analyze a system with two type of failure. The rate of failure is exponential .The system resolved by Boolean algebra. Several measures of availability and MTTF of system are obtained .Some numerical examples, along with graph have been appended at the end to highlight the importance of the result.

Keywords— Gold Extraction System, weibull Distribution, Exponential Distribution , Boolean Algebra

I. INTRODUCTION

Gold has been known and highly valued since pre historic times. It may have been the first metal used by humans for making jewellery and also used in rituals. Approximately 60% of the gold mined today is held by government and central banks. It is presently the most significant and precious metal for international payment. The demand of the gold is increasing day-by-day, therefore, its annual world production has been steadily increasing over the years reaching approximately 2300 metric tonnes. The process of extraction of gold have been studied by many earlier researchers [1]–[2]. Many times, traditional mathematical approaches are tedious to solve the mathematical equations of complex system.

Several methods exist that can be used to improve the system reliability [3]. Reliability is the probability that it performs its purpose within tolerance for the period of time intended under the given operating conditions. Reliability stresses four main components: probability, tolerance, time and operating conditions. [4]Gupta P.P, Agarwal S.C has considered a Boolean Algebra Method for Reliability calculations and again Gupta P.P Kumar Arvind, Reliability and M.T.T.F Analysis of Power Plant. Sharma, Deepankar Sharma, Neelam, Evaluation of some Reliability Parameters for Telecommunication system by Boolean Function Technique [5][6][7][8]. [6][7] Have calculated the reliability of several electronic equipments using various techniques but the method adopted by them lead to cumbersome and tedious calculation. Keeping this fact in view the authors have applied Boolean function technique for the evaluation of various factors of reliability of Gold extraction process.

Amalgamation is an ancient process which involves the alloying of gold particles with metallic mercury to form amalgam and then the separation of the gold from the mercury by heating until the mercury is distilled off. In gravity concentration, it use the principal that all gravity concentrator devices create movement between the gold and host rock particles so that separate the heavy pieces from the lighter pieces of material. Next stage is flotation. It consists of producing a mineral concentrate through the use of chemicals which is followed by intense agitation and air sparging of the agitated ore slurry to produce mineral rich form concentrate. Specific chemicals are added to either float specific minerals or to depress the flotation of other minerals. Advantages of flotation processes are that gold values are generally liberated at a fairly coarse particle size which means that ore grinding costs are minimised. This process is generally used when gold is found in mixture with other metals such as copper, zinc or lead. In heap leaching place crushed or run of mine ore in a pile. The solution of cyanide is distributed upon the top of the pile and the solution percolates down through the pile and leaches out the gold. The gold laden solution drains out from the bottom of the pile and is collected for gold recovery by either carbon absorption or zinc precipitation. The barren solution is then recycled to the pile.

The main oxidation process are of the following:

1. Pressure oxidation:—It utilises oxygen and heat under pressure in a liquid medium to effect oxidation by controlled chemical reaction. The requirement of oxygen overpressure is determined by the mass transfer requirements of the processes and the higher
overpressure tend to shorten the reaction rate and thus the residence time.

2. Bio-oxidation: It uses sulphur consuming bacteria in a water solution to remove sulphur. Advantages include using of air instead of oxygen and normal atmospheric normal conditions. Disadvantages include long retention time due to slow bacterial oxidation rate.

3. Roasting: It uses heat and air to burn away the sulphur from dry ore. It was the standard method for oxidation of sulphur many years ago as it was acceptable to emit large quantities of sulphur dioxide gas into the atmosphere.

4. Chemical oxidation: It uses nitric acid at an ambient pressure and temperature which is now used on a limited basis.

**Gold Extraction Method:** Although new process are being proposed on a regular basis, there have in fact been no dramatic changes in the metallurgical techniques for gold extraction since the introduction of the cyanide process by McArthur and Forrester in 1887. The categories of extraction of gold is shown in block diagram.

**ASSUMPTIONS**

The following assumptions have been associated with this model.

(i) Initially all components are in operable condition.
(ii) Each component is either in good or in fail condition.
(iii) There is no repair facility.
(iv) The state of all system is statistically independent.
(v) The reliability of all component is known in advance.
(vi) The failure time of all component is arbitrary.
(vii) Supply between any two components of the System is fully reliable.

**NOMENCLATURE**

\[ X_i = \text{Gold ore} \]
\[ X_2 = \text{Comminution} \]
\[ X_3 = \text{Gravity concentration} \]
\[ X_4 = \text{Amalgamation} \]
\[ X_5 = \text{Retorting} \]
\[ X_6 = \text{Flotation} \]
\[ X_7 = \text{Regrinding} \]
\[ X_8 = \text{Roasting} \]
\[ X_9 = \text{Bio-Oxidation} \]
\[ X_{10} = \text{Pressure Oxidation} \]
\[ X_{11} = \text{Leaching Process} \]
\[ X_{12} = \text{Extraction Process} \]
\[ X_{13} = \text{Gold Concentrate} \]
\[ X_i^* = \text{negation of } X_i \]
\[ \& = \text{conjunction} \]
\[ || = \text{logical matrix} \]
\[ R_i = \text{reliability of } i^{th} \text{ part of the system}, \]
\[ i = 1, 2, \ldots, 13 \]
\[ R_s = \text{Reliability of the whole system} \]
\[ R_s/R_{se} = \text{reliability of a system as a} \]
Whole when failures follows Weibull/ Exponential time distribution.

**IV. FORMULATION OF MATHEMATICAL MODEL**

By making use of Boolean function technique, the conditions of capability of successful operation of the system in terms of logical matrix are expressed.

\[
\begin{align*}
    x_1 & & x_2 & & x_3 & & x_{13} \\
    x_1 & & x_2 & & x_4 & & x_5 & & x_{13} \\
    x_1 & & x_2 & & x_6 & & x_7 & & x_{11} & & x_{12} & & x_{13} \\
    x_1 & & x_2 & & x_6 & & x_8 & & x_{11} & & x_{12} & & x_{13} \\
    x_1 & & x_2 & & x_6 & & x_9 & & x_{11} & & x_{12} & & x_{13} \\
    x_1 & & x_2 & & x_6 & & x_{10} & & x_{11} & & x_{12} & & x_{13} \\
    x_1 & & x_2 & & x_6 & & x_{11} & & x_{12} & & x_{13} \\
    \cdots (1)
\end{align*}
\]

**V. SOLUTION OF THE MODEL**

By using algebra of logics, we may write equation (1) again as follows

\[
f(x_1, x_2, \ldots, x_{13}) = \left[ (x_1 x_2 x_{13}) \land (x_1 x_2, \ldots, x_{13}) \right] \quad \cdots (2)
\]

Where,

\[
M_1 = |x_3| \quad \cdots (4)
\]

\[
M_2 = |x_4 x_5| \quad \cdots (5)
\]

\[
M_3 = |x_6 x_7 x_{11} x_{12}| \quad \cdots (6)
\]

\[
M_4 = |x_6 x_9 x_{11} x_{12}| \quad \cdots (7)
\]

\[
M_5 = |x_6 x_9 x_{11} x_{12}| \quad \cdots (8)
\]

\[
M_6 = |x_6 x_9 x_{11} x_{12}| \quad \cdots (9)
\]

\[
M_7 = |x_{11} x_{12}| \quad \cdots (10)
\]

Now,

\[
f'(x_1, x_2, \ldots, x_{13}) = \\
\begin{align*}
    M_1 & & M_2 \\
    M_1' & & M_2' & & M_3 \\
    M_1' & & M_2' & & M_3' & & M_4 \\
    M_1' & & M_2' & & M_3' & & M_4' & & M_5 \\
    M_1' & & M_2' & & M_3' & & M_4' & & M_5' & & M_6 \\
    M_1' & & M_2' & & M_3' & & M_4' & & M_5' & & M_6' & & M_7 \\
    \cdots (11)
\end{align*}
\]

\[
M_1' = |x_3'| \quad \cdots (12)
\]

\[
M_2' = \begin{bmatrix} x_4' \\ x_4' \\ x_5' \end{bmatrix} \quad \cdots (13)
\]

\[
M_3' = \begin{bmatrix} x_6' \\ x_6' \\ x_7' \\ x_7' \\ x_{11}' \\ x_{11}' \\ x_{12}' \\ x_{12}' \end{bmatrix} \quad \cdots (14)
\]

\[
M_4' = \begin{bmatrix} x_6' \\ x_6' \\ x_8' \\ x_8' \\ x_{11}' \\ x_{11}' \\ x_{12}' \\ x_{12}' \end{bmatrix} \quad \cdots (15)
\]

\[
M_5' = \begin{bmatrix} x_6' \\ x_6' \\ x_9' \\ x_9' \\ x_{11}' \\ x_{11}' \\ x_{12}' \\ x_{12}' \end{bmatrix} \quad \cdots (16)
\]

\[
M_6' = \begin{bmatrix} x_6' \\ x_6' \\ x_{10}' \\ x_{10}' \\ x_{11}' \\ x_{11}' \\ x_{12}' \\ x_{12}' \end{bmatrix} \quad \cdots (17)
\]

\[
M_7' = \begin{bmatrix} x_{11}' \\ x_{11}' \\ x_{12}' \end{bmatrix} \quad \cdots (18)
\]

Now

\[
M_1'M_2 = |x_3' x_4 x_5| \quad \cdots (19)
\]

\[
M_1'M_2'M_3 = \\
\begin{bmatrix} x_3' & x_4' & x_6 & x_7 & x_{11} & x_{12} \\
    x_3' & x_4' & x_5' & x_6 & x_7 & x_{11} & x_{12} \end{bmatrix} \quad \cdots (20)
\]

\[
M_1'M_2'M_3'M_4 = \\
\begin{bmatrix} x_3' & x_4' & x_6 & x_7 & x_8 & x_{11} & x_{12} \\
    x_3' & x_4' & x_5' & x_6 & x_7 & x_8 & x_{11} & x_{12} \end{bmatrix} \quad \cdots (21)
\]

\[
M_1'M_2'M_3'M_4'M_5 = \\
\begin{bmatrix} x_3' & x_4' & x_6 & x_7 & x_8 & x_{11} & x_{12} \\
    x_3' & x_4' & x_5' & x_6 & x_7 & x_8 & x_{11} & x_{12} \end{bmatrix} \quad \cdots (22)
\]

\[
M_1'M_2'M_3'M_4'M_5'M_6 = \\
\begin{bmatrix} x_3' & x_4' & x_6 & x_7 & x_8' & x_9 & x_{11} & x_{12} \\
    x_3' & x_4' & x_5' & x_6 & x_7 & x_8' & x_9 & x_{11} & x_{12} \end{bmatrix} \quad \cdots (23)
\]
Using all these values in equation (11). One can obtain:

\[ f(x_1, x_2, \ldots, x_{13}) = \]

\[ x_3 \]
\[ x_3' \]
\[ x_4' \]
\[ x_5 \]
\[ x_6' \]
\[ x_7' \]
\[ x_8' \]
\[ x_9' \]
\[ x_10' \]
\[ x_11 \]
\[ x_{12} \]
\[ x_3' \]
\[ x_4' \]
\[ x_5' \]
\[ x_6' \]
\[ x_7' \]
\[ x_8' \]
\[ x_9' \]
\[ x_{10}' \]
\[ x_{11}' \]
\[ x_{12}' \]
\[ x_3' \]
\[ x_4' \]
\[ x_5' \]
\[ x_6' \]
\[ x_7' \]
\[ x_8' \]
\[ x_9' \]
\[ x_{10}' \]
\[ x_{11}' \]
\[ x_{12}' \]

Since eqn(25) is the disjunction of pair wise disjoint conjunctions, therefore, the reliability of the whole system is given by:

\[ R_s = R_1 R_2 R_3 [R_4 + R_4 R_5 - R_4 R_5 + R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_3 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_3 R_6 R_7 R_9 R_{10} R_{11} R_{12}] \]

Where \( R_i \) (i=1, 2, \ldots, 12) is the reliability of the section state, \( x_i \) (i=1, 2, \ldots, 12) respectively.

**SOME PARTICULAR CASES**

Case I): If the reliability of each section of system is same (suppose R) then equation yields:

\[ \bar{x}_{12} \]
\[ R_s = R + R^2 + R^3 + 4R^4 - 2R^5 + 7R^6 - 7R^7 + 3R^8 + R^9 \]

Case ii): When failure rate follows Weibull time distribution: Let \( a, b \) be the failure rate corresponding to section state \( x_i \), for \( i = 1, 2, \ldots, 13 \) then reliability of gold extraction method at an instant ‘s’ is given by

\[ R_{sw}(t) = \exp \{-at\} + \exp \{-2at\} + \exp \{-3at\} + \exp \{-4at\} + 2\exp \{-5at\} + 7\exp \{-6at\} + 7\exp \{-7at\} \]
\[ + 3\exp \{-8at\} + \exp \{-9at\} \]

Case iii): When the failure rate follows exponential distribution

Exponential distribution is nothing but a particular case of Weibull distribution for \( s=1 \) and is very useful for practical problems purpose. Therefore the reliability of considered system as a whole at an instant ‘s’ is expressed as

\[ R_{se} = \frac{e^{-at} + e^{-2at} - e^{-3at} + 4e^{-4at} - 2e^{-5at} + 7e^{-6at} - 7e^{-7at} - 3e^{-8at} + e^{-9at}}{1 + e^{-at} + e^{-2at} + e^{-3at} + e^{-4at} + e^{-5at} + e^{-6at} + e^{-7at} + e^{-8at} + e^{-9at}} \]

The expression for M.T.T.F in this case is given by

\[ M.T.T.F. = \int_0^\infty R(t) \, dt \]
\[ = \frac{1998474}{1} \]

**VI. RESULT**

Let \( a=0.2 \) and \( s=2 \) then comparing Weibull and Exponential distribution with increasing time.
Table 1

<table>
<thead>
<tr>
<th>t</th>
<th>R_{SE}</th>
<th>R_{SW}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>1.875135909</td>
<td>1.592259243</td>
</tr>
<tr>
<td>1</td>
<td>1.943586207</td>
<td>1.943586207</td>
</tr>
<tr>
<td>1.5</td>
<td>1.736525597</td>
<td>1.341279560</td>
</tr>
<tr>
<td>2</td>
<td>1.469762557</td>
<td>0.714410164</td>
</tr>
<tr>
<td>2.5</td>
<td>1.221740951</td>
<td>0.370820985</td>
</tr>
<tr>
<td>3</td>
<td>1.014128995</td>
<td>0.190963165</td>
</tr>
<tr>
<td>3.5</td>
<td>0.847156993</td>
<td>0.093312445</td>
</tr>
<tr>
<td>4</td>
<td>0.714410164</td>
<td>0.042366881</td>
</tr>
<tr>
<td>4.5</td>
<td>0.608565646</td>
<td>0.017720999</td>
</tr>
</tbody>
</table>

Fig-3 Showing reliability in different distribution

Table 2

<table>
<thead>
<tr>
<th>a</th>
<th>M.T.T.F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>1699.444</td>
</tr>
<tr>
<td>0.002</td>
<td>849.722</td>
</tr>
<tr>
<td>0.003</td>
<td>566.481</td>
</tr>
<tr>
<td>0.004</td>
<td>424.861</td>
</tr>
<tr>
<td>0.005</td>
<td>339.888</td>
</tr>
<tr>
<td>0.006</td>
<td>283.240</td>
</tr>
<tr>
<td>0.007</td>
<td>242.777</td>
</tr>
<tr>
<td>0.008</td>
<td>212.430</td>
</tr>
<tr>
<td>0.009</td>
<td>188.827</td>
</tr>
</tbody>
</table>

Fig-4

VII. CONCLUSION

A set of difference differential equations have been obtained by using probability considerations and limiting procedures for various possible systems states of the system. Availability, profit function and M.T.T.F. for the considered system has been obtained. The probabilities of various systems a particular case (when all the repairs follow exponential time distribution) have also been computed to improve practical utility of the model. A numerical example together with its graphical illustration has also been mentioned to highlight important results of the study.

Fig-3 shows that how availability of gold extraction decreases with increase in time.

Fig-4 computes the mean time to failure of system for different values of failure rates. An inspection of the graphs, M.T.T.F. vs failure rate shows that in the beginning M.T.T.F. decreases catastrophically but it decreases approximately at a uniform rate.

REFERENCES

[8] Dhillon B.S., “Reliability Quality and Safety Engineers(Book Style)”, Taylo Francis, U.K.,(2004)
Authors Profile

Mr. A. Chandra is presently pursuing Ph.D. in Mathematics from Amity Institute of Applied Sciences, Amity University (India). He is working on the area of “Reliability modeling”. He did his B.Sc. (H) degree in Mathematics from University of Delhi (India) and M.Sc. Mathematics degree from Himachal Pradesh University (India). He has also been involved in teaching Statistics to the students of Post Graduate programs.

Dr. S. Gupta is an Assistant professor in the field of mathematics in Amity University Noida. She has fifteen years experience of teaching B.sc. and M.sc. students. She is a life member of ISIAM and IAENG. She has published Research papers in various international journals and guided the ph.D students for their research projects. Her main research work focuses on Reliability Modelling.