IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Volume-5, Issue-12 E-ISSN: 2347-2693

Using Reference Point-Based NSGA-II to System Reliability

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Available online at: www.ijcseonline.org

Received: 20/Nov/2017, Revised: 09/Dec/2017, Accepted: 17/Dec/2017, Published: 31/Dec/2017

***Abstract*—** In principle, a multi-objective optimization problem (MOOP) provides a group of non-dominated solutions (popularly known as Pareto-optimal solutions) for the decision maker (DM). A DM is undecidable to claim one of these solutions to be better than another in the absence of any further information. Due to this reason, a DM needs as many Pareto-optimal solutions as possible. Classical optimization methods are unable to produce multiple solutions at a time because of converting the MOOP to a single-objective optimization problem (SOOP). In the past decades, multi-objective evolutionary algorithms (MOEAs) have been developed to be powerful techniques of identifying a complete picture of the Pareto-optimal solutions space, where a DM can select one out of these solutions according to his/her preference. Moreover, a more efficient MOEA can exploit the search in a better position if the DM provides some general views or ideas about the solution in terms of reference points or weights. Reference point based NSGA-II (R-NSGA-II) is such type of an MOEA where DM's assigned reference points are used to search the solutions and its diversity is controlled efficiently. This paper presents the applicability of the R-NSGA-II algorithm to the system reliability design problem. The simulation results show the advantage of R-NSGA-II over NSGA-II.

***Keywords*—** Multi-objective optimization problem (MOOP), Multi-objective evolutionary algorithms (MOEAs), Reference points, System reliability, Pareto-optimal front (POF)

I. INTRODUCTION

In designing phase of a system, we often need to increase the system reliability and reduce its cost simultaneously. Multi-objective formulation of a system design is a better way to represent such problems. Weighted sum method [1] is applied to solve such types of problem. But this method is not a right way and has some demerits such as producing one solution at a time, time-consuming, difficulty in assigning the weights, inability to get the solution in the non-convex region etc. These issues are generally found in most of the classical optimization methods. So, an alternative way is to search for the POF. In the past decades, many MOEAs like PAES [2], NSGA [3], NPGA [4], SPEA [5], NSGA-II [6] etc. have been developed to cope up such issues. Moreover, if the DM provides some views or general idea about the solution, then that information can be exploited by the algorithm in the right direction. R-NSGA-II [7] is such type of algorithm where DM selects a focused solutions space according to his/her preferences. This helps a DM to cover a more convergent part (small part) of the POF. In this way, a DM avoids finding a solution set covering the whole POF [7]. Sometimes a DM gives more than one clue in terms of reference points then multiple regions are obtained. In this case, the search of the solution becomes parallel and gives more effective results. R-NSGA-II also takes care of biased nature of the DM by assigning the weight of one objective more than other. Here, a system design problem is taken into consideration in the framework of R-NSGA-II. The problem is straightforward. In the broadest sense, reliability is defined as a measure of performance of the system. Reliability enhancement is a key feature of the system design. The other features may be conflicting such as cost, weight, volume etc. These are natural resource consumptions where a designer engineer always tries to reduce it from the system with enhancement of reliability simultaneously.

In this paper, reliability and cost are taken as main objectives of system design. The mathematical model of the problem is presented. A numerical example is given to show the performance of R-NSGA-II over NSGA-II. The remaining Sections of the paper are organized as follows: Section II describes the background of the MOOP with some fundamental definitions as well as its optimization algorithm NSGA-II. Section III gives a short description of its related work. Section IV presents a description of the R-NSGA-II algorithm. Section V presents the mathematical model of the problem with an illustrative example. In Section VI, simulation results are shown graphically with its discussion. Section VII gives the conclusion of this paper.

II. Background

A. Multi-objective optimization

The MOOP tackles more than one objective function simultaneously. It is also known as multi-criteria optimization, multi-performance or vector optimization problem. Mathematically, we can define an MOOP as follows [1].

$$\left.\begin{aligned} &\text{Minimize } F(X)=\left[f_1(X), f_2(X),..., f_k(X)\right]^T \\ &\text{subject to } g_i(X)=0, i=1,2,...,m_e; \\ &\qquad g_i(X)\geq 0, i=m_e+1, m_e+2,...,m; \\ &\qquad x_j^{(L)}\leq x_j \leq x_j^{(U)}; j=1,2,...,n, \end{aligned}\right\} \quad (1)$$

where $k\geq 2$ is the number of objectives; m is the total number of constraints; m_e is the number of equality constraints; $X=\left[x_1,x_2,...,x_n\right]^T$ is n dimensional decision vector from the feasible region $\Omega\subseteq \mathbb{R}^n$ (Euclidean n -space); objective functions $f_p(X), p=1,2,...,k,$ where $f_p:\Omega\to\mathbb{R}$ and the constrained functions $g_i(X)$, where $g_i:\Omega\to\mathbb{R}$; $F(X)$ is called a multi-objective vector or criterion vector; $x_j^{(L)}$ and $x_j^{(U)}$ are the lower and upper bounds of the decision variable x_j respectively.

If all f_p's and g_i's are linear then the problem is called a multi-objective linear programming problem (MOLPP), otherwise, it is called a multi-objective nonlinear programming problem (MONLPP). Two Euclidean spaces are considered in an MOOP as Decision space and Objective space. For every point X in the decision space, there exists a point $F(X)=\left[f_1(X), f_2(X),..., f_k(X)\right]^T$ in the objective space. Therefore, there is a mapping between n-dimensional solution vector and k-dimensional objective vector. It is obvious that the approach of the SOOP is not directly applicable to MOOP. Due to this reason, a classification of solutions is given in terms of Pareto optimality. From MOOP given in (1), we can define the following definitions as follows:

Definition 1 (Pareto dominance). A solution vector X^1 Pareto-dominates another solution vector X^2 denoted as $X^1\succ X^2$ iff [1]

- $f_p(X^1)\leq f_p(X^2)\forall p=1,2,...,k,$
- $f_q(X^1)< f_q(X^2)$ for at least one $q\in\{1,2,...,k\}$, $p\neq q$.

If there does not exist such solutions which Pareto-dominate to X^1 then solution vector X^1 is called a non-dominated solution.

Definition 2 (Pareto-optimal set). A set of non-dominated solutions $P=\left\{X^* \mid \neg\exists X \; s.t. X\succ X^*\right\}$ is called a Pareto-optimal set [8].

Definition 3 (Pareto-optimal solution). A point $X^*\in\Omega$ is called a Pareto-optimal solution if there does not exist another point $X\in\Omega$ such that $f_p(X)\leq f_p(X^*)\forall p=1,2,...,k$ and $f_q(X)< f_q(X^*)$ for at least one $q\in\{1,2,...,k\}$, $p\neq q$ [8].

Definition 4 (Pareto-optimal front). The set of vectors in the objective space that is mapped to elements from a Pareto-optimal set under F,
i.e., $POF=\left\{F(X^*) \mid \neg\exists X \; s.t. X\succ X^*\right\}$ [8].

Two important goals in a multi-objective optimization [1]:
First goal: To find a set of solutions as close as possible to the POF.
Second goal: To find a set of solutions as diverse homogeneous as possible.

The first goal indicates the convergence of solutions near the POF while the second goal refers to get a uniformly spaced set of solutions indicating an adequate exploration of the search space and not losing valuable information.

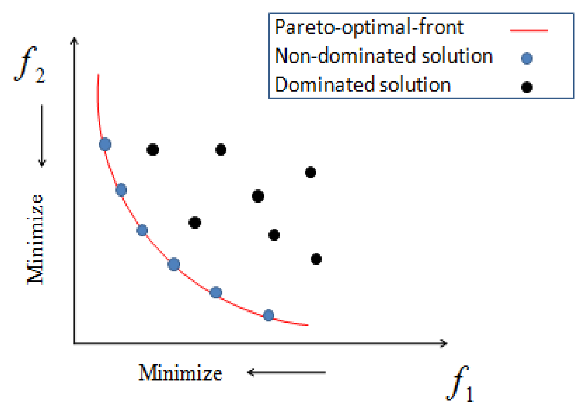

Figure 1. A schematic representation of Pareto-optimal front

B. Non-dominated Sorting Genetic Algorithm-II (NSGA-II)

Non-dominated sorting genetic algorithm (NSGA) was initially suggested by Srinivas and Deb [3]. It uses Goldberg's domination criterion [9] to assign ranks for the solutions and utilization of fitness sharing for maintaining the diversity in the solution set. It has some difficulty in regarding computational complexity, non-elitist approach and highly dependent on the parameters of fitness sharing. Deb et al. [6] extended this algorithm in form of NSGA-II by giving some new features like fast non-dominated sorting, crowding distance, and comparison operator.

NSGA-II assigns a rank for solutions employing non-dominated sorting procedure (see Fig. 2) and emphasizes good solutions throughout this algorithm. The overall complexity governed by this process is $O(kN^2)$, where k and N denote the no. of objectives and population size respectively [6].

For maintaining the diversity in the solution set, NSGA-II calculates the crowding distance (see Fig. 3) of each solution. It is basically defined as those solutions that contain the same rank. A partial order comparison operator is applied to determine a better solution between two solutions. According to this operator, if both the solutions belong to the same rank then preference is given to the solution that contains a higher crowding distance value. A higher crowding distance value gives the lesser crowded region and vice versa [6]. The NSGA-II procedure is given in Fig. 4.

The pseudo code of NSGA-II algorithm is given as follows:

Step 1. Initializing randomly a parent population P_0 of size *N*. Setting $k = 0$.

Step 2. Assigning fitness (rank) according to non-domination level and crowded-comparison operator.

Step 3. **while** k < number of maximum generation **do**

- (i) Creating an offspring population Q_k of size N applying reproduction, crossover, and mutation.
- (ii) Combining via $R_k = P_k \cup Q_k$.
- (iii) Sorting on R_k and classifying them into non-dominated fronts (Pareto-front) $PF_i, i = 1, 2, ...,$ etc.
- (iv) Setting a new population $P_{k+1} = \phi$ and $i = 1$.
  while the parent population size $|P_{k+1}| + |PF_i| < N$ **do**
  - (i) Calculating the crowding distance of PF_i
  - (ii) Adding the i^{th} non-dominated front PF_i to the parent population P_{k+1}.
  - (iii) $i = i + 1$.

  end while
- (iv) Sorting the PF_i using the crowding distance based comparison operator.
- (v) Filling the parent population P_{k+1} with the first $N - |P_{k+1}|$ solutions of PF_i.
- (vi) Generating the offspring population Q_{k+1}.
- (vii) Setting $k = k + 1$.

end while

Step 4. Collecting the non-dominated solutions in the vector *P*.

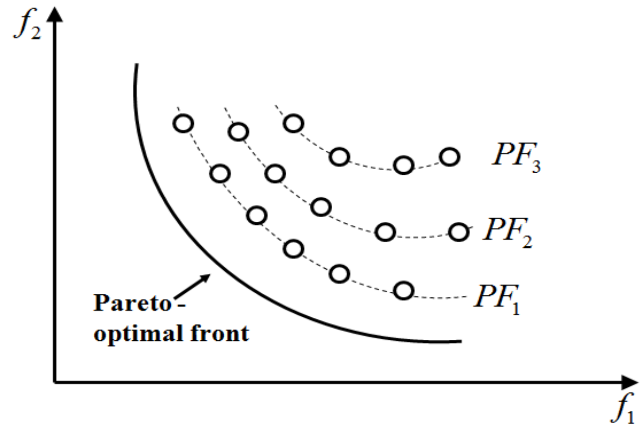

Figure 2. Non-dominated sorting of a population

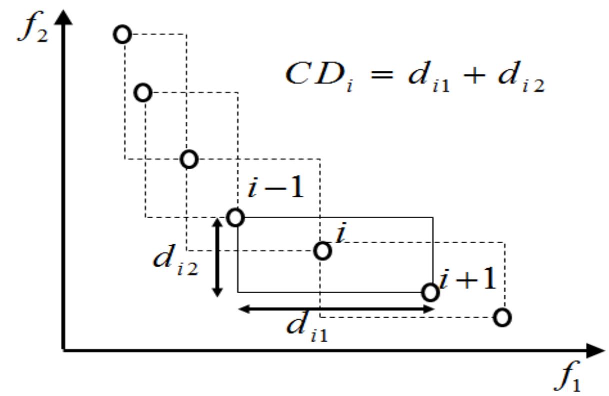

Figure 3. Crowding distance evaluation of a solution

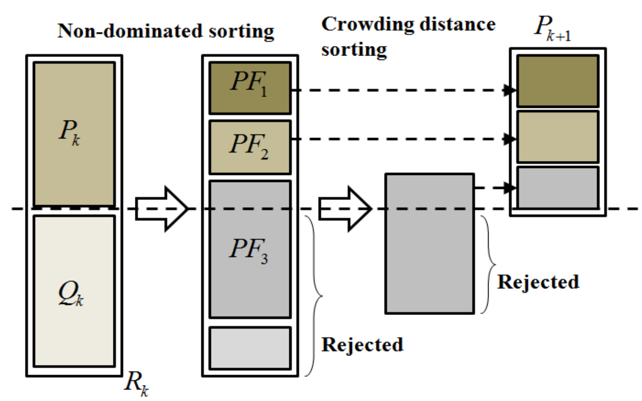

Figure 4. Evaluation cycle of NSGA-II

III. RELATED WORK

The second generation MOEA, NSGA-II was applied to multi-objective reliability optimization problems by Salazar et al. [10]. The competency of NSGA-II to solve unconstrained and constrained reliability optimization problems over the existing approaches have been demonstrated.

Kishore et al. [11] show the applicability of NSGA-II in a series system reliability problem. Kishore et al. [12] also proposed an interactive approach for fuzzy multi-objective reliability optimization problem based on NSGA-II.

Wang et al. [13] solved the redundancy allocation problem (RAP) in parallel series under a number of constraints using NGSA-II and compared with existing heuristic methods for single objective optimization problems.

Safari [14] proposed a variant of NSGA-II in solving a multi-objective RAP.

Khalili-Damghani et al. [15] proposed a decision-support system for multi-objective RAPs.

Taboada et al. [16] proposed practical solutions of system reliability design problems by combining NSGA algorithm with k-mean clustering technique.

IV. Reference Point-Based NSGA-II (R-NSGA-II)

The R-NSGA-II [7] is an MOEA. This is an extension of the NSGA-II algorithm with some specified preferences of the DM in terms of reference points. NSGA-II algorithm has difficulty in producing a preferred or smaller set of Pareto-optimal solutions. In the practical point of view, a DM should allow in concentrating those regions of the POF which are of his/her interest. This type of MOEA has an advantage over interactive classical optimization methods where we get only a single solution based on the preferences assigned by the DM. Getting a single solution near the desired region of the front [7] is not an ideal property. A DM is interested in knowing the characteristics of optimum and near-optimum solutions with help of given clue rather than looking for a single solution. Therefore, a number of solutions near the reference point are more reliable over a single solution in making a better decision. Moreover, if a DM provides multiple regions of such interests simultaneously then search becomes more effective and parallel towards a preferred solution. These characteristics are well handled by the R-NSGA-II algorithm. Here, a DM is allowed to give one or more reference points in order to attract the search for new solutions. In Fig. 5, the flowchart of the R-NSGA-II algorithm is presented. The selection procedure of R-NSGA-II in the flow chart (marked as sub-process) is different from original NSGA-II. The updates of NSGA-II are given in the following steps [7].

Step 1. R-NSGA-II applies non-dominated sorting process of solutions slightly different from NSGA-II. Firstly, the normalized Euclidean distance of every solution is calculated with respect to each reference point. The normalized Euclidean distance D_{XR} from solution X to the reference point R is defined as

$$D_{XR} = \sqrt{\sum_{i=1}^{k} \left(\frac{f_i(X) - R_i}{f_i^{\max} - f_i^{\min}} \right)^2} \qquad (2)$$

This distance is called the *preference distance*.

Step 2. After performing such computations in Step 1, the minimum preference distance of the solutions is assigned a rank one. Similarly, next minimum preference distance is assigned a rank of two and so on. In this way, all solutions are sorted in ascending order of distance. Then, solutions having smaller rank are preferred in the tournament selection to form new population from a combined population of parent and offspring.

Step 3. To control the diversity of the solutions, R-NSGA-II applies the ϵ-clearing idea to the niching operator in replace of crowding distance. ϵ-clearing idea (value gap) is responsible to maintain the diversification among all solutions. The ϵ-value is known as the tolerance or precision for the objective values specified by the DM. The tolerance or precision is decided by the DM according to the given conditions.

Firstly, a solution is randomly chosen from the non-dominated solution set. By considering the selected solution as a central point, R-NSGA-II calculates the distance of adjacent solutions to this point until the radius of ϵ. In other words, it calculates the sum of the normalized difference in objective values of ϵ or less from the chosen solution [7]. At this point, in order to discard the solutions in ϵ -vicinity of the selected solutions, R-NSGA-II assigns them a large distance value. In this way, only one solution (central solution) is selected and other solutions are discarded. Thereafter, the algorithm selects another unconsidered solution from the remaining solutions in the non-dominated set. The process is repeated and continues until reaching all solutions.

The procedure described above gives equal importance of solutions nearest to each reference point and finds multiple regions of interest simultaneously in a single simulation run.

Moreover, ϵ-based selection strategy (also known as ϵ-dominance strategies) makes sure a spread of solutions near the preferred regions. In other words, reference point approach is equivalent to a weight vector emphasizing each objective function equally as $w_i = 1/k$.

In order to make biasedness towards some objectives more than others, an appropriate weight vector is supplied by the DM to each reference point rather than giving priority in emphasizing solutions with the shortest Euclidean distance from a reference point. For this purpose, a shortest weighted

Euclidean distance from the reference point is calculated as follows [7].

$$D_{XR}^{w} = \sqrt{\sum_{i=1}^{k} w_i \left(\frac{f_i(X) - R_i}{f_i^{\max} - f_i^{\min}} \right)^2} \tag{3}$$

where $f_i^{\min}$ and $f_i^{\max}$ are the minimum and maximum values of i^{th} the objective. This distance is also used to get a set of preferred solutions in the non-convex region.

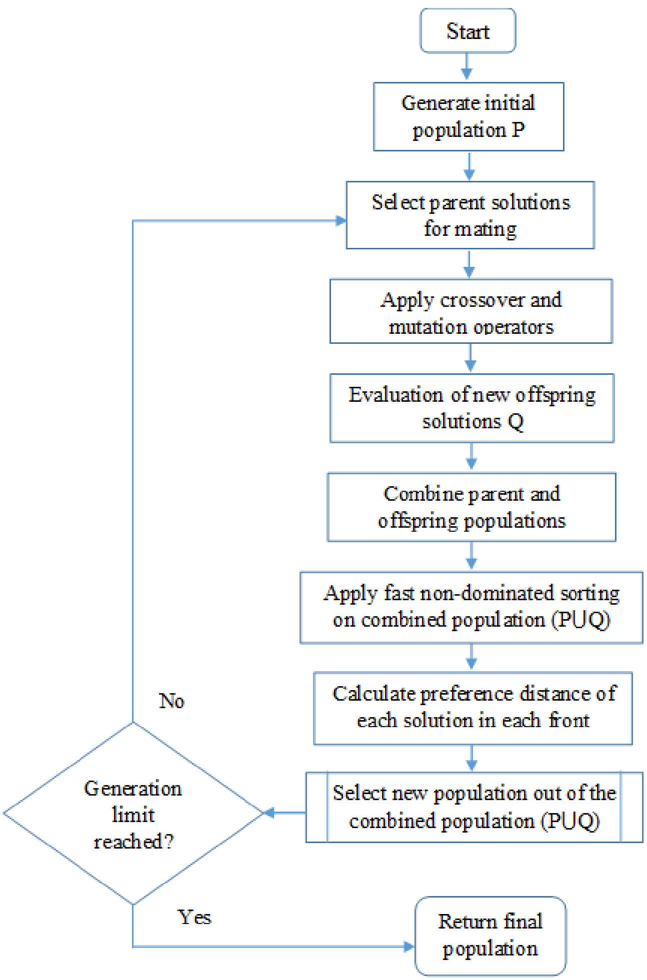

Figure 5. Flowchart of R-NSGA-II algorithm

V. Mathematical Model of the Problem

In practical situations, the problem of system reliability is constructed as a typical non-linear programming problem with nonlinear cost. A design engineer wants to increase the system reliability and decrease its cost simultaneously. The cost of reliability is a monotonically increasing function of reliability [17]. Therefore, an MOOP is modeled by taking system reliability R_s and system cost C_s simultaneously as objectives given by

$$\text{Maximize } R_s(r_1, r_2, \ldots, r_n) = \begin{cases} \prod_{i=1}^{n} r_i & \text{for series system} \\ \text{or} & \\ 1 - \prod_{i=1}^{n} (1 - r_i) & \text{for parallel system} \\ \text{or} & \\ \text{combination of series and parallel system} & \end{cases} \tag{4}$$

$$\text{Minimize } C_S(r_1, r_2, \ldots, r_n) = \sum_{i=1}^{n} C_i(r_i) \tag{5}$$

$$\text{subject to } r_{i,\min} \le r_i \le 1, \text{ for } i = 1, 2, \ldots, n, \tag{6}$$

where n is the total number of components in the system and $r_{i,\min}$ is a minimum value for the i^{th} component.

A. An illustrative example

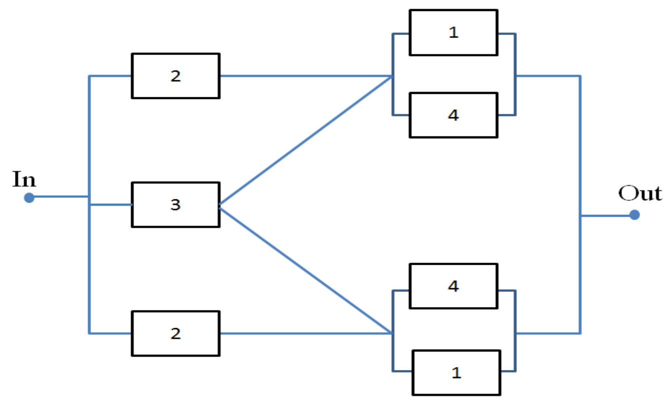

Figure 6. Life-support system in a space capsule

In Fig. 6, the system configuration of the complex system (Life-support system in a space capsule) [18] is shown. The system needs a single path to its success, possesses two redundant subsystems, each subsystem connects with two redundant components 1 and 4. Each of the redundant subsystems connects in series with component 2 and the resultant pair of series-parallel arrangement forms two equal paths. In order to back up for the pair, component 3 enters as a third path. This problem forms a continuous nonlinear optimization problem and consists of four components, each having component reliability $r_i, i = 1,2,3,4$.

$$\text{Maximize } R_s = 1 - r_3\left[(1-r_1)(1-r_4)\right]^2 - (1-r_3)\left[1 - r_2\{1-(1-r_1)(1-r_4)\}\right]^2 \tag{7}$$

$$\text{Minimize } C_S = 2\sum_{i=1}^{4} K_i r_i^{\alpha_i} \tag{8}$$

$$\text{subject to } 0.5 \le r_i \le 1, i = 1, 2, 3, 4. \tag{9}$$

In other words, the problem (7) to (9) can be written as

$$\text{Minimize}(Q_s, C_s),\ Q_s = 1 - R_s \tag{10}$$

$$\text{subject to } 0.5 \le r_i \le 1, i = 1,2,3,4 \tag{11}$$

where vectors of coefficients K_i and α_i are $K = \{100, 100, 200, 150\}$ & $\alpha = \{0.6, 0.6, 0.6, 0.6\}$ respectively.

VI. Simulation Results and Discussion

To solve the MOOP (7) to (9), the list of parameters of R-NSGA-II is given in Table 1. We have used the Intermediate crossover [1] and Gaussian mutation [1] operator to generate offspring population. The best POF is observed on the basis of tuning of the parameters. The R-NSGA-II algorithm has been programmed in MATLAB R2010a and runs in MS window environment on the PC which has *intel core*^{TM} *i3* Duo processor with 2.40 GHz and 2GB RAM.

Table 1. Parameter settings for R-NSGA-II

Parameters	Value/Type
Population size	200
Number of generations	500
Number of design variables	4
Number of objectives	2
Lower bounds	[0.5 0.5 0.5 0.5]
Upper bounds	[1 1 1 1]
Population Initialization	Random
Selection strategy	Binary Tournament
Crossover probability	0.9
Mutation probability	0.25

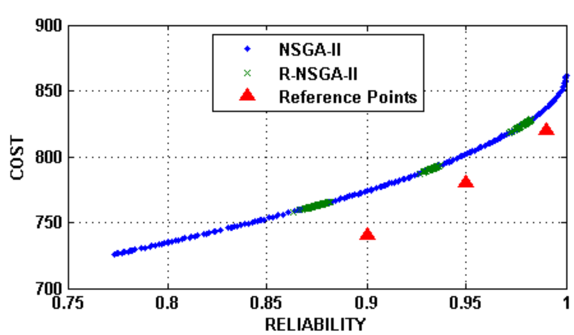

Figure 7. POF with reference points as (0.90, 740), (0.95, 780) and (0.99, 820)

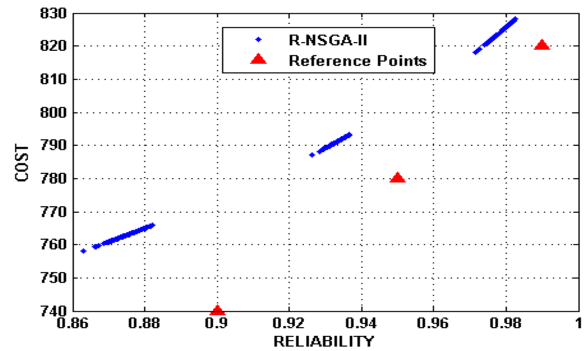

Figure 8. POF at $\in = 0.001$ (diversity parameter)

Fig. 7 shows the advantage of R-NSGA-II over NSGA-II. Three reference points are taken in keeping the view of the DM's choice. The reference points are (0.90, 740), (0.95, 780) and (0.99, 820). R-NSGA-II has the tendency to achieve the most favorable parts of the POF. In Figs. 8 and 9, the effect of epsilon in obtaining a varied spread of preferred solutions has been shown.

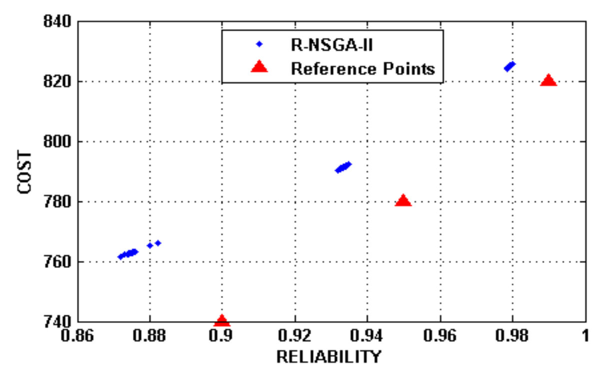

Figure 9. POF at $\in = 0.0001$ (diversity parameter)

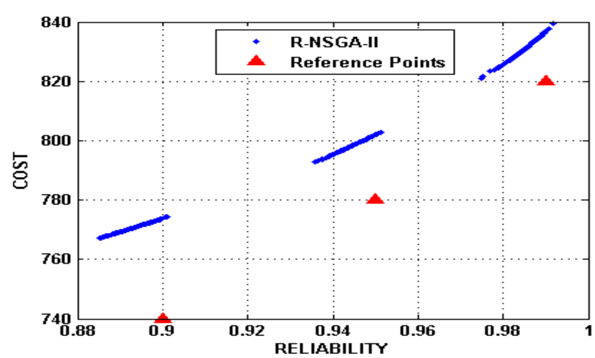

Figure 10. Biased preferred solutions (POF) with weight vector (0.8, 0.2)

In Figs. 10, 11, 12, the effect of the weight vector in R-NSGA-II at ϵ =0.001 has been shown. If the DM is excited in biasing one objective more than another then a suitable weight vector can be applied to these reference points. In keeping the views of the DM, three weight vectors such as (0.8, 0.2), (0.5, 0.5) and (0.8, 0.2) have been considered between reliability and cost of the system. This is similar to the classical scalarization approach.

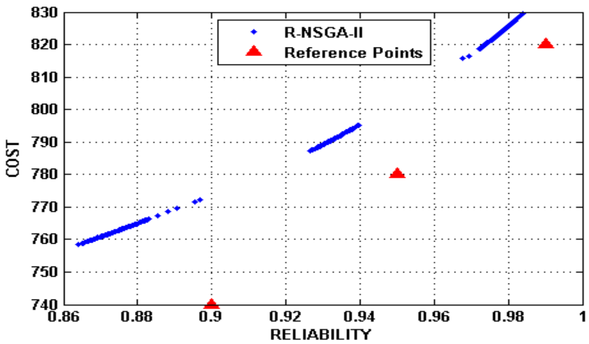

Figure 11. Non-biased preferred solutions (POF) with weight vector (0.5, 0.5)

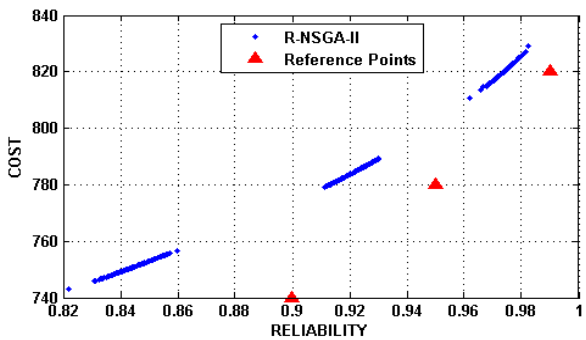

Figure 12. Biased preferred solutions (POF) with weight vector (0.2, 0.8)

VII. CONCLUSION AND FUTURE SCOPE

In this piece of work, the significance of R-NSGA-II algorithm is shown in reliability based system design problem. A mathematical model of the problem with two conflicting objectives reliability vs cost is presented and then a numerical example of a complex system is given for illustration. Three reference points are chosen according to the choice of the DM. In order to tackle the biased behavior of the DM of one objective more than another, three weight vectors (0.8, 0.2), (0.5, 0.5) and (0.2, 0.8) are taken into consideration. The behavior of the POF has been successfully demonstrated in each case. The effect of epsilon in a varied spread of preferred solutions has been successfully shown. We find that epsilon is decreased from the original value 0.001 to 0.0001 then the creation of Pareto-optimal solutions gets diminished. The main advantage of using R-NSGA-II is that we get multiple solutions (Pareto-optimal solutions) in a single simulation run with each reference point. The concept of reference point methodology can be used in other MOEA approaches and implemented in many decision-making fields such as economics, management, engineering etc.

ACKNOWLEDGMENT

The first author acknowledges to MHRD, Govt. of India for providing the financial grant.

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