

Modeling of Blood Flow Through Artery With Magnetic Effects

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Abstract— The most important aspiration of present study is to make a mathematical and simulation modeling for magnetic effect of blood within undersized artery. Power law fluid characterizes the non-Newtonian quality of blood. The dynamical functioning of the blood flow is affected by the occurrence of the magnetic effects. The problem is worked out with analytical procedures with facilitate of boundary conditions and consequences are put on show graphically for different flow uniqueness like pressure drop, blood velocity, shear stress, etc. For the justification of mathematical model, the computation outcomes are compared with consequences from published text. In this article blood flow uniqueness are calculated for a precise set of values of the diverse factors concerned in the model examination and presented graphically. Few obtained outcome indicate that the flow characteristics in converging region, diverging region, and nontapered region are efficiently influenced by the occurrence of magnetic electrically field and justify inclination of artery and magnetic area respectively.

Keywords— Non-Newtonian flow, Artery, Magnetic field, Shear stress, Velocity.

I. INTRODUCTION

Under usual circumstances, blood flow in the man circulatory system relies upon the pumping act of the heart and this produce a pressure gradient over all the artery and vein network system. Pressure gradient is having two mechanisms, first of which is regular said to be non-fluctuating and the other is fluctuating known as pulsatile. The main purpose to get knowledge of blood flow by arteries is understanding significance in a lot of heart diseases mainly in atherosclerosis. The regular thing of blood flow is uneasy due to a few odd developments like stenosis in the lumen of the human being's artery in pulse rate etc.

In present time, the outcome of magnetic area on the flow of viscous liquid in the course of a regular porous liquid has been the subject of frequent submissions. RBC is a main biomagnetic matter, and the blood flow may be subjective by the magnetic field. In broad manner, biological systems are concerned by a submission of exterior magnetic field on blood flow, through human being arterial structure. The occurrences of the stationary magnetic field serve to rise in the resistance of flowing blood.

A Study on Shape optimization in stable blood flow with numerical investigation of non-Newtonian Blood has done by Abraham [1]. It is suggested that the Performance modeling & investigation of blood flows in elastic artery by Kumar [2]. It's told that finite element Galerkin's program for flow in blood pipes with magnetic effects by Kumar [3]. It's showed that unsteady reaction of non Newtonian blood flow by a stenosed narrow blood vessel in electrically field by Chakravarty [4]. Human artery MHD pulsatile flow of liquid influenced by periodic body acceleration calculated by Das [5]. Pulsatile flow of paired stress fluid through porous system influenced by periodic body acceleration & electrically flow field was investigated by Rathod [6]. An Analytical result has found of 2-dimensional system of fluid flow with changeable viscosity through an indented blood artery due to LDL consequence in the occurrence of magnetic field by Singh [7]. A good work has done on modeling of blood fluid flow resistance for an atherosclerotic artery with manifold stenosis & Post stenotic dilatations by Wong [8]. A lot of work has done on performance, modeling, mechanical behavior & investigation of blood flow in small vessels with magnetic effects by Kumar [9,10]. The work concerned to slip effects on the unstable MHD pulsatile fluid flow through porous system in an artery influenced by body acceleration has done by Eldesoky [11]. Performance modeling & mechanical behavior of blood vessel in the survival of magnetic effects has studied by Anil [12]. Individual study has done on flow properties of the blood through an inclined narrowed porous blood artery system with mild stenosis influenced by inclined magnetic field by Srivastava [13]. Modeling of magnetohemodynamics in the stenosed arteries in diabetic and anemic models

was studied by Aiman [14]. A good information on blood flow in human arterial System was provided by Blessy [15]. Velocity report of unsteady blood flow through an inclined circular pipe with magnetic field was studied by Mwanthi [16]. In this paper we have put introduction, material and methods, results, conclusions and references on I, II, III, IV and V number respectively.

II. MATERIAL AND METHODS

Blood is understood as an electrically carrying out fluid when concerned to a magnetic field in the case of electromagnetic force is produced and an electrical current flows as a consequence. The complicatedness consists of the result of the transient leading momentum, Navier - Stokes relations and the electrical Maxwell's associations for the magnetic field is

$$D = \delta(E + u \times c) \quad (1)$$

In the equation (1), E is electrically field intensity, δ is electrically conductivity, c is said to magnetic flux strength and v is velocity of the liquid. In the momentum relation the electromagnetic strength F_n is included and clarify as

$$F_n = D \times c = \delta(u \times c) \times c \quad (2)$$

Navier Stokes relations for blood flow when Lorentz force is implemented

$$\rho_0 \left(\frac{\partial u}{\partial t} + u \cdot \nabla u \right) = \delta(u \times c) \times c - \nabla p + \mu_0 \nabla^2 u \quad (3)$$

Where ρ_0 , μ_0 and p are density, viscosity and pressure respectively.

The continuity equation in a CPC system used to develop in the reason of an incompressible blood flow

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0 \quad (4)$$

The general relation of axial momentum

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} = \frac{1}{\rho_0} \left[-\frac{\partial p}{\partial z} + \left\{ \left[\frac{1}{r} \frac{\partial (r \delta_{rz})}{\partial r} + \frac{\partial (\delta_{zz})}{\partial z} \right] \right\} \right] + g_z \quad (5)$$

The general relation of radial momentum

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} = \frac{1}{\rho_0} \left[-\frac{\partial p}{\partial r} + \left\{ \left[\frac{1}{r} \frac{\partial (r \delta_{rr})}{\partial r} + \frac{\partial (\delta_{rz})}{\partial z} \right] \right\} \right] + g_r \quad (6)$$

In the above equation $u_z(r, z)$ and $u_r(r, z)$ represents axial and radial speed. P is called pressure and ρ_0 is the density of blood. The sample of wall shear pressure in an artery for an inlet velocity is similar for the entire Non Newtonian and Newtonian models.

Here δ is said to be stress tensor. Fluid behavior components are

$$\delta_{rr} = -p + 2\mu \frac{\partial u_r}{\partial r}, \quad \delta_{zz} = -p + 2\mu \frac{\partial u_z}{\partial z} \quad \text{and} \quad \delta_{rz} = \delta_{zr} = \mu \left[\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right]$$

The pressure gradient emerging in equation (6) is given by

$$-\frac{\partial p}{\partial z} = E_0 + E_1 \cos \omega t, \quad t > 0 \quad (7)$$

$$G = e_0 \cos(\omega t + \xi) \quad (8)$$

E_0 is called an amplitude of the pressure gradient and E_1 is considered an amplitude of pulsatile flow providing grow to systolic and diastolic pressure & $\omega = 2\pi f$ where f is the pulse regularity and t is consumed time.

The blood flow equations are derived with the assistance of given boundary conditions. In the channel, stress free environment are implemented and at border wall no slip situation is supposed.

$$u_r = u_z = 0 \quad \text{at} \quad r = R_0 \quad (9)$$

At the middle line, the blood velocity is finite.

$$\frac{\partial u_z}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad (10)$$

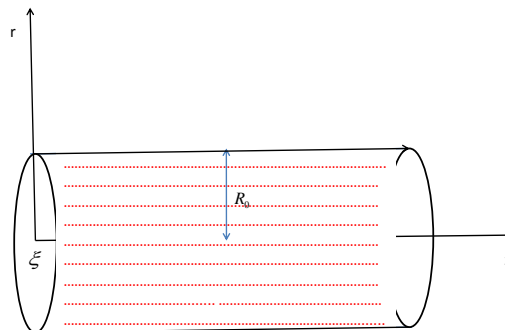


Figure 1. Construction of blood flow through artery model

The governing equation is:

$$\rho_0 \frac{\partial u_r}{\partial t} = E_0 + E_1 \cos \omega t + e_0 \rho_0 \cos(\omega t + \xi) + \left(\mu_0 + \mu_1 \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - Ku - \sigma_0 B^2 u \quad (11)$$

Here $K = \frac{\mu_0}{k}$, where $u(r, t)$ is required velocity in axial direction, ρ_0 and μ_0 are density & viscosity of blood fluid respectively, μ_1 is elastic-viscosity coefficient of the fluid, K is the permeability constraint and r is radial coordinate axe.

The dimensionless quantities are:

$$u^* = \frac{u}{\omega R_0}, \quad r^* = \frac{r}{R_0}, \quad E_0^* = \frac{R_0}{\mu_0 \omega} E_0, \quad E_1^* = \frac{R_0}{\mu_0 \omega} E_1, \quad z^* = \frac{z}{R_0}, \quad t^* = t \omega, \quad B = \frac{\omega_2}{\omega_1}, \quad e_0' = \frac{\rho_0 R_0}{\mu_0 \omega} e_0$$

Equation (11) becomes

$$\alpha^2 \frac{\partial u}{\partial t} = E_0 + E_1 \cos t + e_0 \cos(bt + \xi) + \left(1 + \beta \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - du \quad (12)$$

Here $d = H^2 + M^2$, $\alpha = R_0 \sqrt{(\omega \rho_0 / \mu_0)}$ is a Womersley measure parameter, $H = R_0 B \sqrt{(\sigma_0 / \mu_0)}$ is consider Hartmann number and $k = \frac{E}{R_0}$ is said to be Knudsen number.

Initial and boundary conditions are:

$$u(r, z, 0) = \left(\frac{E_0 + E_1}{d} \right) \left[1 - \frac{I_0 \sqrt{dr}}{I_0 \sqrt{d}} \right] \quad \text{Where } \begin{cases} u(1, t) = 0 \\ u(0, t) = \text{finite} \end{cases}$$

If $f(r)$ convinced the Dirichlet's circumstances in interval $(0, 1)$, now finite Hankel transforms is

$$f^*(\lambda_n) = \int_0^1 r f(r) J_0(r \lambda_n) dr \quad (13)$$

Where λ_n are the roots of equation $J_0(r) = 0$ and

$$f(r) = 2 \sum_{n=1}^{\infty} f^*(\lambda_n) \frac{J_0(r \lambda_n)}{J_1^2(r \lambda_n)} \quad (14)$$

By using Laplace transform equation (12) becomes

$$\alpha^2 p \bar{u} - \alpha^2 u(r, 0) = \frac{E_0}{p} + \frac{E_1 p}{p^2 + 1} + \frac{e_0(p \cos \xi - b \sin \xi)}{p^2 + b^2} + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) [\bar{u} + \beta p \bar{u} - \beta u(r, 0) - d \bar{u}] \quad (15)$$

Apply finite Hankel change

$$\bar{u}^*(\lambda_n, p) = \frac{J_1(\lambda_n)}{\lambda_n} \left[\begin{aligned} & \frac{E_0}{\lambda_n + d} \left(\frac{1}{p} - \frac{1}{p+h} \right) + \frac{E_1(\lambda_n^2 + d)}{(\lambda_n + d)^2 (\alpha^2 + \lambda_n^2 \beta)^2} \\ & \left\{ \frac{-1}{p+h} + \frac{p}{p^2 + 1} + \frac{\alpha^2 + \lambda_n^2 \beta}{(\lambda_n^2 + d)(p^2 + 1)} \right\} + \frac{e_0(\lambda_n^2 + d) \cos \xi}{(\lambda_n^2 + d)^2 + (\alpha^2 + \lambda_n^2 \beta) b^2} \\ & \left\{ \frac{-1}{p+h} + \frac{p}{p^2 + b^2} + \frac{(\alpha^2 + \lambda_n^2 \beta) b^2}{(\lambda_n^2 + d)(p^2 + b^2)} \right\} - \frac{e_0 b \sin \xi (\alpha^2 + \lambda_n^2 \beta)^2}{(\lambda_n^2 + d)^2 + b^2 (\alpha^2 + \lambda_n^2 \beta)^2} \\ & \left\{ \frac{1}{p+h} - \frac{p}{p^2 + b^2} + \frac{\lambda_n^2 + d}{(p^2 + b^2)(\alpha^2 + \lambda_n^2 \beta)^2} \right\} + \frac{E_0 + E_1}{(\lambda_n^2 + d)} \frac{1}{(p+h)} \end{aligned} \right] \quad (16)$$

$$\text{Where } h = \frac{\lambda_n^2 + d}{\alpha^2 + \lambda_n^2 \beta} \quad (17)$$

The Laplace transforms and Hankel mathematical inversion of equation (16) provides the final result as

$$u(r, t) = 2 \sum_{n=1}^{\infty} \frac{J_0(\lambda_n * r)}{\lambda_n J_1(\lambda_n)} \left\{ \begin{aligned} & \frac{E_0}{\lambda_n^2 + d} + \frac{E_1 [(\lambda_n^2 + d) \cos t + (\alpha^2 + \beta \lambda_n^2) \sin t]}{(\lambda_n^2 + d)^2 + (\alpha^2 + \lambda_n^2 \beta)^2} + \\ & \frac{e_0 [(\lambda_n^2 + d) \cos(bt + \xi) + (\alpha^2 + \beta \lambda_n^2) \sin(bt + \xi)]}{(\lambda_n^2 + d)^2 + b^2 (\alpha^2 + \lambda_n^2 \beta)^2} \end{aligned} \right\} + e^{-ht} \left[\begin{aligned} & \frac{E_0}{(\lambda_n^2 + d)} + \frac{E_1(\lambda_n^2 + d)}{(\lambda_n^2 + d)^2 + (\alpha^2 + \beta \lambda_n^2)^2} - \frac{E_0 + E_1}{\lambda_n^2 + d} \\ & + \frac{e_0 [(\lambda_n^2 + d) \cos \xi + (\alpha^2 + \lambda_n^2 \beta) \sin \xi]}{(\lambda_n^2 + d)^2 + b^2 (\alpha^2 + \lambda_n^2 \beta)^2} \end{aligned} \right] \quad (18)$$

Then derivation for the flow velocity $Q = 2\pi \int_0^1 r u dr$

$$Q(r,t) = 4 \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} \left\{ \begin{aligned} & \frac{E_0}{\lambda_n^2 + d} + \frac{E_1[(\lambda_n^2 + d) \cos t + (\alpha^2 + \beta \lambda_n^2) \sin t]}{(\lambda_n^2 + d)^2 + (\alpha^2 + \lambda^2 \beta)^2} \\ & + \frac{e_0[(\lambda_n^2 + d) \cos(bt + \xi) + (\alpha^2 + \beta \lambda_n^2) \sin(bt + \xi)]}{(\lambda_n^2 + d)^2 + b^2(\alpha^2 + \beta \lambda_n^2)^2} \end{aligned} \right\} \quad (19)$$

$$+ e^{-ht} \left[\begin{aligned} & \frac{E_0}{\lambda^2 + d} + \frac{E_1(\lambda_n^2 + d)}{(\lambda_n^2 + d)^2 + (\alpha^2 + \beta \lambda_n^2)^2} - \frac{E_0 + E_1}{\lambda^2 + d} \\ & + \frac{e_0[(\lambda_n^2 + d) \cos \xi + (\alpha^2 + \lambda_n^2 \beta) \sin \xi]}{(\lambda_n^2 + d)^2 + b^2(\alpha^2 + \lambda_n^2 \beta)^2} \end{aligned} \right]$$

Equally the mathematical derivation for fluid acceleration $F(r,t) = \frac{\partial u_r}{\partial t}$ (20)

$$F(r,t) = 2 \sum_{n=1}^{\infty} \frac{J_0(\lambda_n^* r)}{\lambda_n J_{1k}(\lambda_n)} \left\{ \begin{aligned} & \frac{E_0}{\lambda_n^2 + d} - \frac{E_1[(\lambda_n^2 + d) \sin t - (\alpha^2 + \beta \lambda_n^2) \cos t]}{(\lambda_n^2 + d)^2 + (\alpha^2 + \lambda^2 \beta)^2} \\ & - \frac{e_0 b[(\lambda_n^2 + d) \sin(bt + \xi) - (\alpha^2 + \beta \lambda_n^2) \cos(bt + \xi)]}{(\lambda_n^2 + d)^2 + (\alpha^2 + \beta \lambda_n^2)^2} \end{aligned} \right\} \quad (21)$$

$$+ \frac{1}{h} e^{-ht} \left[\begin{aligned} & \frac{E_0}{(\lambda^2 + d)} + \frac{E_1(\lambda_n^2 + d)}{(\lambda_n^2 + d)^2 + (\alpha^2 + \beta \lambda_n^2)^2} - \frac{E_0 + E_1}{\lambda^2 + d} \\ & + \frac{e_0[(\lambda_n^2 + d) \cos \xi + (\alpha^2 + \lambda_n^2 \beta) \sin \xi]}{(\lambda_n^2 + d)^2 + b^2(\alpha^2 + \lambda_n^2 \beta)^2} \end{aligned} \right]$$

III. RESULT

The velocity $u(r,t)$, $F(r,t)$ and $Q(r,t)$ are acquired in equation (18), (19) and (21) have been described in figures 2 and 3 by plotting r in opposition to u for differ values of Hartmann number M , for pressure gradient E_0 , shape is constructed for r in opposition to $F(r,t)$ for differ values of M . Figure 2 explains that $u(r,t)$ reduces with raise in M , shape 3 explains that $u(r,t)$ decreases with increase in E_0 . Figure 4 specifies that there Variation of acceleration of blood for differ values of Hartmann number M and the quantity of this retardation is reducing with the raise in M . Figure 5 described that there is a raise in the velocity as Womersley parameter α is raised.

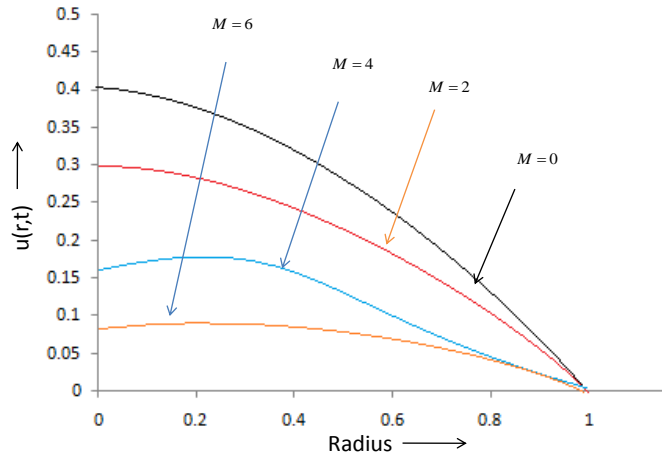


Figure 2. Variation of velocity profile for differ values of M
 $E_0 = 2, E_1 = 4, e_0 = 3, b = 1, H = 2, \alpha = 1, t = 1$

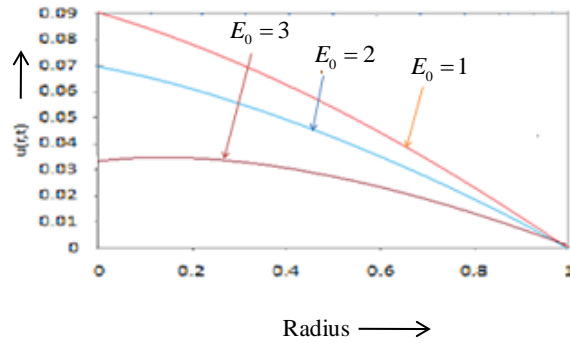


Figure 3. Variation of velocity profile for differ values of Pressure Gradient
 $E_1 = 1, e_0 = 3, b = 1, H = 2, \alpha = 1$

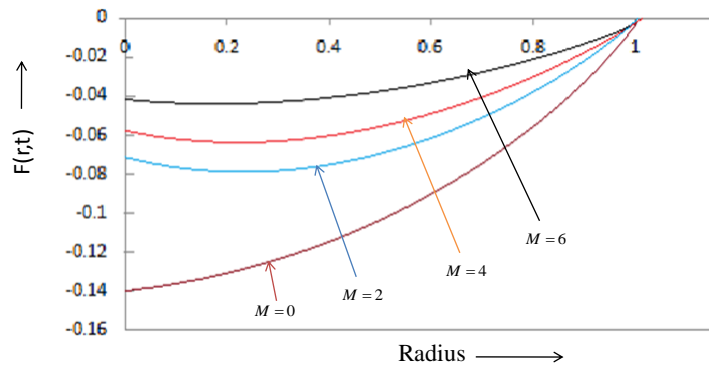


Figure 4. Variation of acceleration of blood for differ values of M
 $E_0 = 2, E_1 = 4, e_0 = 3, b = 1, H = 2, \alpha = 1, t = 0.5$

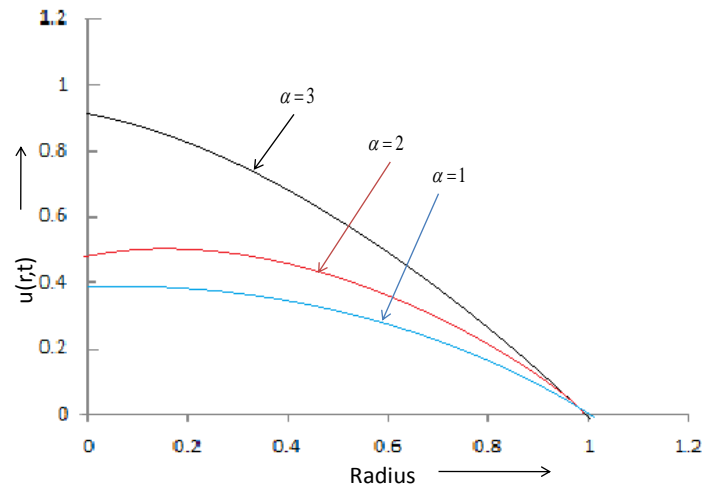


Figure 5. Variation of velocity profile with Womersley parameter

$$E_0 = 1, E_1 = 1, e_0 = 0, b = 0, Ha = 2, k \times 0.1 = 1, t = 1$$

IV. CONCLUSION

The current algorithm is reasonable and competent, having a pointed convergence. The result show that magnetic field reduces the stress. In the current mathematical replica the blood flow under the control of periodic body acceleration, variation in velocity for Hartmann number, pressure gradient as well as Womersley parameter with the magnetic effect has been examined. The corresponding expression for flow rate, fluid acceleration & shear stress has also obtained. It is probable that an appropriate understanding of interactions of body acceleration with blood flow may guide to a therapeutic use of controlled body acceleration. The desirable effects of different vibrations on different branches of the body. The current work calculated that magnetic effect of blood flow by undersized vessel, which is of great curiosity for the use of medical sciences. Magnetic field is influencing by the flow of blood by undersized vessel, which is beneficial for the problem like blood pressure, hypertension etc.

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Authors Profile

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